Strong Performers and Successful Reformers in Education

Lessons from PISA 2012 for the United States

Programme for International Student Assessment

OECD

Strong Performers and Successful Reformers in Education

LESSONS FROM PISA 2012
FOR THE UNITED STATES

OECD
BETTER POLICIES FOR BETTER LIVES

Please cite this publication as:
OECD (2013), *Lessons from PISA 2012 for the United States,* Strong Performers and Successful Reformers in Education, OECD Publishing.
http://dx.doi.org/10.1787/9789264207585-en

ISBN 978-92-64-20757-8 (print)
ISBN 978-92-64-20758-5 (PDF)

Photo credits:
© Ministry of Education of The People's Republic of China.

Corrigenda to OECD publications may be found on line at: *www.oecd.org/publishing/corrigenda*.

Foreword

In 2010, U.S. President Barack Obama launched one of the world's most ambitious education reform agendas. Entitled "Race to the Top", the agenda encourages U.S. states to adopt internationally benchmarked standards and assessments as a framework within which it can prepare students for success in college and the workplace; recruit, develop, reward, and retain effective teachers and principals; build data systems that measure student success and inform teachers and principals how they can improve their practices; and turn around their lowest-performing schools.

Alongside the *Race to the Top* initiative, a state-led effort established a single set of clear education standards for kindergarten through 12th grade (K-12) in English-language arts and mathematics that states voluntarily adopt. Some 46 states and the District of Columbia have become members of the initiative and are adopting the standards.

Three years on from the launch of *Race to the Top*, where does the United States stand when compared internationally? Will effective implementation of the Common Core State Standards lead to improvement in U.S. performance as measured by international benchmarks? The OECD Programme for International Student Assessment (PISA) provides the world's most extensive and rigorous set of international surveys of the knowledge and skills of secondary school students. It allows one to compare countries on measures such as their average learning outcomes, their share of low-performing schools, the extent to which socio-economic background shapes learning outcomes, and how consistently their schools deliver high-quality outcomes.

This volume follows on from the report *Strong Performers and Successful Reformers in Education: Lessons from PISA for the United States*, which was published when the PISA 2009 results became available in 2010. It provides an update on the performance of the United States in PISA and delves more deeply into the data to expose the strengths and weaknesses of U.S. students in mathematics, the main focus of PISA 2012.

This volume is the result of a collaborative effort between the OECD, international and U.S. national experts and was financed by a grant from the Hewlett Foundation. The report was prepared under the responsibility of the Early Childhood and Schools Division of the OECD Directorate for Education and Skills, principally Michael Davidson, under the oversight of Andreas Schleicher, Deputy Director for Education and Skills.

The principal authors of the chapters in this report were Michael Davidson (Chapters 1 and 2), Werner Blum and his team from Humbolt University (Chapter 3), and Jason Zimba, a lead writer of the Common Core State Standards for Mathematics (Chapter 4). The OECD PISA team provided information, analysis and advice to support the PISA analysis in this volume, and the material in Chapter 2 is almost entirely drawn from Volumes I-IV of the *PISA 2012 Results*, the lead authors of which are Sophie Vayssettes, Pablo Zoido, Francesco Borgonovi and Miyako Ikeda. Elisabeth Villoutreix co-ordinated the steps towards publication, editorial support was provided by Sally Hinchcliffe, Cassandra Davis and Marilyn Achiron, and administrative support was provided by Juliet Evans, Diana Tramontano and Claire Chetcuti.

Table of Contents

Executive Summary

This report situates the performance in PISA of 15-year-olds in the United States against global patterns and trends. It goes beyond the aggregate-level analysis to analyze the strengths and weaknesses of U.S. students on different types of mathematics tasks. It also reviews the relationship with the Common Core State Standards for Mathematics (CCSSM) in order to help connect results from PISA to what the United States aspires to teach in classrooms and to inform teaching practices that can help to improve performance.

KEY FINDINGS FROM PISA

The United States remains in the middle of the rankings

Among the 34 OECD countries, the United States performed below average in mathematics (rank 26[1]) and around the average in reading (rank 17[2]) and science (rank 21[3]) in the 2012 PISA assessment of 15-year-olds, and the trend data show no significant changes in these performances over time.

Just over one-quarter (26%) of 15-year-olds in the United States do not reach the PISA baseline Level 2 of mathematics proficiency. This percentage is higher than the OECD average of 23% and has remained unchanged since 2003. By contrast, in Canada, Hong Kong-China, Korea, Shanghai-China and Singapore, the proportion of poor performers is around 10% or less (Level 2 on the PISA mathematics scale can be considered a baseline level of proficiency at which students begin to demonstrate the skills that will enable them to participate effectively and productively in life).

At the other end of the performance scale, the United States also has a below-average share of top performers in mathematics. It does slightly better in reading and science where the proportion of top performers is around the OECD average. Only 2% of students in the United States reach the highest level (Level 6) of performance in mathematics, compared with an OECD average of 3% and up to 31% of students in Shanghai-China.

Equality of opportunity

Socio-economic disadvantage has a notable impact on student performance in the United States: 15% of the variation in student performance in the United States is explained by students' socio-economic status, similar to the OECD average. This contrasts with less than 10% in a number of countries/economies including Finland, Hong Kong-China, Japan and Norway.

Unlike around half of OECD countries, in the United States there is no significant difference between advantaged and disadvantaged schools in terms of student-teacher ratios or the proportion of mathematics teachers who have university-level qualifications.

In the United States, 5% of students can be considered resilient, meaning that they are among the 25% most socio-economically disadvantaged students but nevertheless perform much better than would be predicted by their socio-economic status. This is below the OECD average (7%) and is only around one-third of the proportion observed in Hong Kong-China, Macao-China, Shanghai-China and Viet Nam.

The learning environment

Some 30% of 15-year-old students in the United States reported that they had arrived late for school at least once in the two weeks prior to the PISA test, slightly below the OECD average of 35%, and 20% of students in the United States reported that they had skipped a day of school in the previous two weeks, above the OECD average of 15%.

Compared with students in other countries, 15-year-olds in the United States view teacher-student relations relatively positively. Still, schools in the United States that perform better tend to have more positive teacher-student relations, even after accounting for student and school characteristics.

Schools whose principals reported that teachers' behavior negatively affects learning to a great extent also tend to be those whose principals reported that their teachers' morale is low. The United States is one of the countries where this relationship is particularly strong. Similarly, the United States shows one of the strongest correlations between schools with a student population that is predominantly socio-economically disadvantaged and a more negative school disciplinary climate.

The organization of schooling

Across OECD countries on average, schools with more autonomy over curricula and assessments tend to perform better than schools with less autonomy when they are part of school systems with more accountability arrangements and greater teacher-principal collaboration in school management.

Around three-quarters of students in the United States attend schools that compete with at least one other school for enrolment (similar to the OECD average), yet there is no evident cross-country relationship between the degree of competition among schools and student performance.

Assessment and accountability

Over 80% of students in the United States attend schools whose principals reported that achievement data are posted publicly (the OECD average is 45%), and virtually all students are in schools whose principals reported that achievement data are tracked over time by an administrative authority (the OECD average is 72%).

Resources

Higher expenditure on education is not highly predictive of better PISA mathematics scores among OECD countries. For example, the United States and the Slovak Republic both score 481 points in mathematics, but the United States' cumulative expenditure per student is more than double that of the Slovak Republic.

In the United States, as well as in many other countries, schools where a larger share of principals reported that teacher shortages hinder learning tend to show lower performance. The United States is one of a group of countries where advantaged and disadvantaged schools show particularly wide differences in the extent of teacher shortages.

Principals in advantaged schools in the United States tend to have much more positive views of the adequacy of material resources than principals in disadvantaged schools.

Across countries on average, students who had attended pre-primary education tend to perform better at the age of 15 than those who had not attended pre-primary education. While in almost all countries this performance advantage remains constant after accounting for socio-economic background, this is not the case in the United States.

STRENGTHS AND WEAKNESSES IN MATHEMATICS

The proportion of students who answered each of the 84 mathematics items correctly varied markedly. The average success rate across all items was 43.8%, below the OECD average (47.5%) and even further below the averages of the five countries chosen as comparators: Canada, Germany, Korea, the Netherlands and Shanghai-China. However, U.S. students performed nearly as well as students in Shanghai-China, better than the OECD average, and better than up to four of the other reference countries in several items. Conversely, U.S. students performed far below the OECD average and below the reference countries some other items; these items constitute the relative weaknesses of the United States.

Analyses of the items on which United States students performed relatively well indicate that the strengths in their mathematical competence lies in reading data directly from tables and diagrams; simple handling of data from tables and diagrams; and handling directly manageable formulae. In terms of weaknesses, the analysis suggests that U.S.

students struggle with tasks requiring students to: use and apply the number π; establish a mathematical model of a given real world situation; genuinely interpret real world aspects; and reason in a geometric context.

An implication of the findings is that much more focus is needed on higher-order activities, such as those involved in mathematical modeling (understanding real-world situations, transferring them into mathematical models, and interpreting mathematical results), without neglecting the basic skills needed for these activities.

RELATIONSHIP WITH THE COMMON CORE STATE STANDARDS FOR MATHEMATICS

With most U.S. states having adopted the Common Core State Standards for Mathematics (CCSSM) as their state mathematics standard, the report examines the relationship between these standards and PISA mathematics. The analysis shows some commonality between the PISA framework and items and the CCSSM, and classifies all PISA 2012 mathematics items according to dimensions of the CCSSM.

The analysis suggests that it is intuitively plausible that faithful implementation of the CCSSM would improve PISA results. Presenting high school students with better modeling problems – and testing these skills through assessments – would be an important step in achieving this.

Notes

1. Though rank 26 is the best estimate, due to sampling and measurement error the rank could be between 23 and 29.

2. Though rank 17 is the best estimate, due to sampling and measurement error the rank could be between 14 and 20.

3. Though rank 21 is the best estimate, due to sampling and measurement error the rank could be between 17 and 25.

1

PISA as a Yardstick for Educational Success

More and more countries are looking beyond their own borders for evidence of the most successful and efficient policies and practices. Over the past decade, the OECD Programme for International Student Assessment (PISA) has become the world's premier yardstick for evaluating the quality, equity and efficiency of school systems. This chapter introduces PISA and sets the scene for situating the PISA performance of 15-year-olds in the United States against global patterns and trends.

SITUATING PISA AS A YARDSTICK FOR EDUCATIONAL SUCCESS

Equipping citizens with the skills necessary to achieve their full potential, participate in an increasingly interconnected global economy, and ultimately convert better jobs into better lives is a central preoccupation of policy makers around the world. Results from the OECD's recent Survey of Adult Skills (OECD, 2013a) show that highly skilled adults are twice as likely to be employed and almost three times more likely to earn an above-median salary than poorly skilled adults. In other words, poor skills severely limit people's access to better-paying and more rewarding jobs. Highly skilled people are also more likely to volunteer, see themselves as actors rather than as objects of political processes, and are more likely to trust others. Fairness, integrity and inclusiveness in public policy thus all hinge on the skills of citizens.

Furthermore, the ongoing economic crisis has only increased the urgency of investing in the acquisition and development of citizens' skills – both through the education system and in the workplace. At a time when public budgets are tight and there is little room for further monetary and fiscal stimulus, investing in structural reforms to boost productivity, such as education and skills development, is key to future growth. Indeed, investment in these areas is essential to support the recovery as well as to address long-standing issues, such as youth unemployment and gender inequality.

In this context, more and more countries are looking beyond their own borders for evidence of the most successful and efficient policies and practices. Indeed, in a global economy, success is no longer measured against national standards alone, but against the best-performing and most rapidly improving education systems. Over the past decade, the OECD Programme for International Student Assessment (PISA) has become the world's premier yardstick for evaluating the quality, equity and efficiency of school systems. But the evidence base that PISA has produced goes well beyond statistical benchmarking. By identifying the characteristics of high-performing education systems PISA allows governments and educators to identify effective policies that they can then adapt to their local contexts.

The results from the PISA 2012 assessment, which was conducted at a time when many of the 65 participating countries and economies were grappling with the effects of the economic crisis, reveal wide differences in education outcomes, both within and across countries. Using the data collected in previous PISA rounds, we have been able to track the evolution of student performance over time and across subjects. Of the 64 countries and economies with comparable data, 40 improved their average performance in at least one subject. Top performers such as Shanghai in China and Singapore were able to further extend their lead, while countries like Brazil, Mexico, Tunisia and Turkey achieved major improvements from previously low levels of performance.

Some education systems have demonstrated that it is possible to secure strong and equitable learning outcomes at the same time as achieving rapid improvements. Of the 13 countries and economies that significantly improved their mathematics performance between 2003 and 2012, three also show improvements in equity in education during the same period, and another nine improved their performance while maintaining an already high level of equity – proving that countries do not have to sacrifice high performance to achieve equity in education opportunities.

Nonetheless, PISA 2012 results show wide differences between countries in mathematics performance. The equivalent of almost six years of schooling, 245 score points, separates the highest and lowest average performances of the countries that took part in the PISA 2012 mathematics assessment. The difference in mathematics performances within countries is even greater, with over 300 points – the equivalent of more than seven years of schooling – often separating the highest- and the lowest-achieving students in a country. Clearly, all countries and economies have excellent students, but few have enabled all students to excel.

The report also reveals worrying gender differences in students' attitudes towards mathematics: even when girls perform as well as boys in mathematics, they report less perseverance, less motivation to learn mathematics, less belief in their own mathematics skills, and higher levels of anxiety about mathematics. While the average girl underperforms in mathematics compared with the average boy, the gender gap in favour of boys is even wider among the highest-achieving students. These findings have serious implications not only for higher education, where young women are already under-represented in the science, technology, engineering and mathematics fields of study, but also later on, when these young women enter the labour market. This confirms the findings of the OECD Gender Strategy, which identifies some of the factors that create – and widen – the gender gap in education, labour and entrepreneurship. Supporting girls' positive attitudes towards and investment in learning mathematics will go a long way towards narrowing this gap.

PISA 2012 also finds that the highest-performing school systems are those that allocate educational resources more equitably among advantaged and disadvantaged schools and that grant more autonomy over curricula and assessments to individual schools. A belief that all students can achieve at a high level, and a willingness to engage all stakeholders

in education – including students, through such channels as seeking student feedback on teaching practices – are hallmarks of successful school systems.

PISA is not only an accurate indicator of students' abilities to participate fully in society after compulsory school, but also a powerful tool that countries and economies can use to fine-tune their education policies. There is no single combination of policies and practices that will work for everyone, everywhere. Every country has room for improvement, even the top performers. That's why the OECD produces this triennial report on the state of education across the globe: to share evidence of the best policies and practices and to offer our timely and targeted support to help countries provide the best education possible for all of their students. With high levels of youth unemployment, rising inequality, a significant gender gap and an urgent need to boost growth in many countries, we have no time to lose. The OECD stands ready to support policy makers in this challenging and crucial endeavour.

For the United States, as for other countries, the results from the OECD Survey of Adult Skills (PIAAC) provides a new context in which the results from PISA can be examined. The Adult Skills Survey showed:

- Larger proportions of adults in the United States than in other countries have poor literacy and numeracy skills, and the proportion of adults with poor skills in problem solving in technology-rich environments is slightly larger than the average, despite the relatively high educational attainment among adults in the United States.
- Socio-economic economic background has a stronger impact on adult literacy skills in the United States than in other countries. Black and Hispanic adults are substantially over-represented in the low-skilled population.
- Literacy skills are linked not only to employment outcomes, but also to personal and social well-being. In the United States, the odds of being in poor health are four times greater for low-skilled adults than for those with the highest proficiency – double the average across participating countries.

But the results from the Survey of Adult Skills also showed the importance of developing key skills through school in order to provide a strong foundation for skills development later in life. The results show that, overall, there is a reasonably close correlation between countries' performance across the successive PISA assessments and the proficiency of the corresponding age cohorts in literacy and numeracy in the Survey of Adult Skills. Countries which performed well in PISA in a given year (e.g. 2000) tend to show high performance among the corresponding age cohort (e.g. 27-year-olds) in the Survey of Adult Skills and vice versa. This suggests that, at the country level, the reading and mathematics proficiency of an age cohort in PISA is a reasonably good predictor of the cohort's subsequent performance in literacy and numeracy as it moves through post-compulsory education and into the labour market.

STRUCTURE OF THIS REPORT

The purpose of this report is to situate the performance of 15-year-olds in the United States in PISA against the global patterns and trends. But it aims to go beyond the aggregate-level analysis that has so far been published in the PISA 2012 international reports, to give an analysis of student performance on individual mathematics test items in order to reveal students' strengths and weaknesses. Considering this also in the context of the relationship between PISA and the Common Core State Standards for Mathematics (CCSSM) can help connect these results to what the United States aspires to teach in classrooms and help inform teaching practices that can support performance improvement.

The remainder of this introductory chapter gives a brief account of what PISA is, its approach and what it measures. Chapter 2 provides an in-depth analysis of the performance of the United States on PISA, contrasting the performance of United States' 15-year-olds in mathematics, reading and science with that of students in other countries and the factors associated with quality and equity of outcomes.

Chapter 3 then looks in more detail at the strengths and weaknesses of students in the United States in the PISA 2012 mathematics assessment. This is done by examining the success rates of students at the item level of the test, compared with the success rates of students across OECD countries on average and in comparison with five comparator or reference sytems. The five countries chosen for comparisons with the United States are: two top-performing Asian countries, Shanghai-China and Korea, two European countries performing significantly above the OECD average, the Netherlands and Germany, and one of United States' neighboring countries, Canada.

The final chapter examines the extent to which the PISA mathematics assessment relates to the the United States Common Core State Standards for Mathematics that a number of its states have adopted. It asks whether faithful implementation of the Common Core State Standards is likely to improve the United States' performance on PISA. The chapter provides an initial investigation into this by seeking to understand, in mathematical terms, how CCSSM relates to the PISA measures and vice versa.

WHAT IS PISA?

"What is important for citizens to know and be able to do?" That is the question that underlies the triennial survey of 15-year-old students around the world known as the Programme for International Student Assessment (PISA). PISA assesses the extent to which students near the end of compulsory education have acquired the key knowledge and skills that are essential for full participation in modern societies. The assessment, which focuses on reading, mathematics, science and problem solving, does not just ascertain whether students can reproduce knowledge; it also examines how well students can extrapolate from what they have learned and apply that knowledge in unfamiliar settings, both in and outside of school. This approach reflects the fact that modern economies reward individuals not for what they know, but for what they can do with what they know.

PISA is an ongoing program that offers insights for education policy and practice, and that helps monitor trends in students' acquisition of knowledge and skills across countries and in different demographic subgroups within each country. PISA results reveal what is possible in education by showing what students in the highest-performing and most rapidly improving education systems can do. The findings allow policy makers around the world to gauge the knowledge and skills of students in their own countries in comparison with those in other countries, set policy targets against measurable goals achieved by other education systems, and learn from policies and practices applied elsewhere. While PISA cannot identify cause-and-effect relationships between policies/practices and student outcomes, it can show educators, policy makers and the interested public how education systems are similar and different – and what that means for students.

PISA's unique features include its:

- **policy orientation**, which links data on student learning outcomes with data on students' backgrounds and attitudes towards learning and on key factors that shape their learning, in and outside of school, in order to highlight differences in performance and identify the characteristics of students, schools and education systems that perform well;
- **innovative concept of "literacy"**, which refers to students' capacity to apply knowledge and skills in key subjects, and to analyse, reason and communicate effectively as they identify, interpret and solve problems in a variety of situations;
- **relevance to lifelong learning**, as PISA asks students to report on their motivation to learn, their beliefs about themselves, and their learning strategies;
- **regularity**, which enables countries to monitor their progress in meeting key learning objectives; and
- **breadth of coverage**, which, in PISA 2012, encompasses the 34 OECD member countries and 31 partner countries and economies.

WHAT DOES THE PISA 2012 SURVEY MEASURE?

The PISA 2012 survey focused on mathematics, with reading, science and problem solving as minor areas of assessment. For the first time, PISA 2012 also included an assessment of the financial literacy of young people, which was optional for countries.

For PISA, mathematics proficiency means the capacity of individuals to formulate, employ and interpret mathematics in a variety of contexts. The term describes the capacities of individuals to reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. Mathematics literacy is not an attribute that an individual either has or does not have; rather, it is a skill that can be developed over a lifetime.

The 2012 survey is the fifth round of assessments since PISA began in 2000, and the second, after the 2003 survey, that focused on mathematics. As such, PISA 2012 provides an opportunity to evaluate changes in student performance in mathematics since 2003, and to view those changes in the context of policies and other factors.

For the first time, PISA 2012 included an optional computer-based assessment of mathematics. Specially-designed PISA questions are presented on a computer, and students respond on the computer, although they can also use pencil and paper as they think through the test questions.

Box 1.1 A test the whole world can take

PISA is now used as an assessment tool in many regions around the world. It was implemented in 43 countries and economies in the first assessment (32 in 2000 and 11 in 2002), 41 in the second assessment (2003), 57 in the third assessment (2006) and 75 in the fourth assessment (65 in 2009 and 10 in 2010). So far, 65 countries and economies have participated in PISA 2012.

In addition to OECD member countries, the survey has been or is being conducted in:

East and Southeast Asia: Himachal Pradesh-India, Hong Kong-China, Indonesia, Macao-China, Malaysia, Shanghai-China, Singapore, Chinese Taipei, Tamil Nadu-India, Thailand and Viet Nam.

Central, Mediterranean and Eastern Europe, and Central Asia: Albania, Azerbaijan, Bulgaria, Croatia, Georgia, Kazakhstan, Kyrgyzstan, Latvia, Liechtenstein, Lithuania, the Former Yugoslav Republic of Macedonia, Malta, Moldova, Montenegro, Romania, the Russian Federation and Serbia.

The Middle East: Jordan, Qatar and the United Arab Emirates.

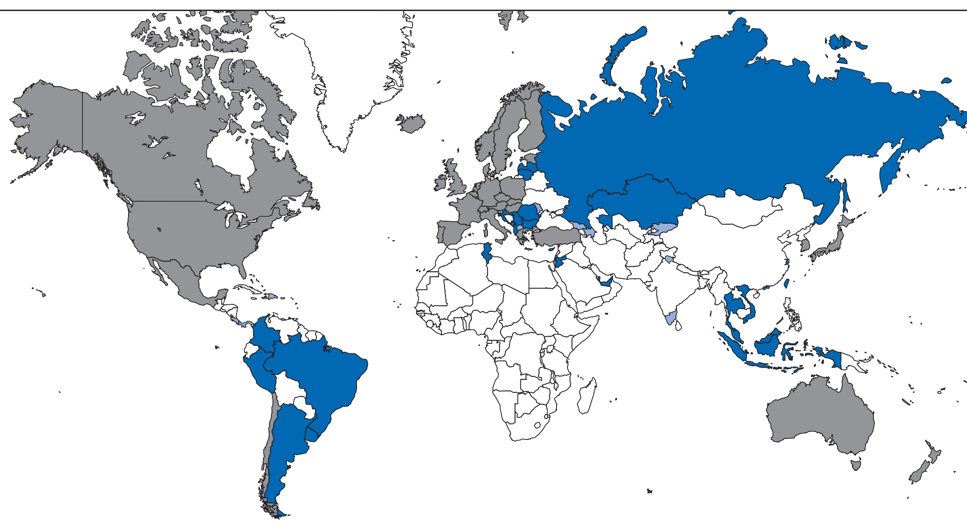
Central and South America: Argentina, Brazil, Colombia, Costa Rica, Netherlands-Antilles, Panama, Peru, Trinidad and Tobago, Uruguay and Miranda-Venezuela.

Africa: Mauritius and Tunisia.

Decisions about the scope and nature of the PISA assessments and the background information to be collected are made by leading experts in participating countries. Considerable efforts and resources are devoted to achieving cultural and linguistic breadth and balance in assessment materials. Since the design and translation of the test, as well as sampling and data collection, are subject to strict quality controls, PISA findings are considered to be highly valid and reliable.

■ Figure 1.1 ■

Map of PISA 2012 countries and economies



OECD countries

Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States

Partner countries and economies in PISA 2012

Albania, Argentina, Brazil, Bulgaria, Colombia, Costa Rica, Croatia, Cyprus[1, 2], Hong Kong-China, Indonesia, Jordan, Kazakhstan, Latvia, Liechtenstein, Lithuania, Macao-China, Malaysia, Montenegro, Peru, Qatar, Romania, Russian Federation, Serbia, Shanghai-China, Singapore, Chinese Taipei, Thailand, Tunisia, United Arab Emirates, Uruguay, Viet Nam

Partner countries and economies in previous cycles

Azerbaijan, Georgia, Himachal Pradesh-India, Kyrgyzstan, Former Yugoslav Republic of Macedonia, Malta, Mauritius, Miranda-Venezuela, Moldova, Panama, Tamil Nadu-India, Trinidad and Tobago

Box 1.2 **Key features of PISA 2012**

The content

- The PISA 2012 survey focused on mathematics, with reading, science and problem solving as minor areas of assessment. For the first time, PISA 2012 also included an assessment of the financial literacy of young people, which was optional for countries.
- PISA assesses not only whether students can reproduce knowledge, but also whether they can extrapolate from what they have learned and apply their knowledge in new situations. It emphasises the mastery of processes, the understanding of concepts, and the ability to function in various types of situations.

The students

- Around 510 000 students completed the assessment in 2012, representing about 28 million 15-year-olds in the schools of the 65 participating countries and economies.

The assessment

- Paper-based tests were used, with assessments lasting a total of two hours for each student. In a range of countries and economies, an additional 40 minutes were devoted to the computer-based assessment of mathematics, reading and problem solving.
- Test items were a mixture of multiple-choice items and questions requiring students to construct their own responses. The items were organised in groups based on a passage setting out a real-life situation. A total of about 390 minutes of test items were covered, with different students taking different combinations of test items.
- Students answered a background questionnaire, which took 30 minutes to complete, that sought information about themselves, their homes, and their school and learning experiences. School principals were also given a 30-minute questionnaire that covered the school system and the learning environment. In some countries and economies, optional questionnaires were distributed to parents, who were asked to provide information on their perceptions of and involvement in their child's school, their support for learning in the home, and their child's career expectations, particularly in mathematics. Countries could choose two other optional questionnaires for students: one asked students about their familiarity with and use of information and communication technologies, and the second sought information about their education to date, including any interruptions in their schooling and whether and how they are preparing for a future career.

REPORTING RESULTS FROM PISA 2012

The results of PISA 2012 are presented in six volumes.

Volume I, *What Students Know and can Do: Student Performance in Mathematics, Reading and Science* summarises student performance in mathematics in PISA 2012 and examines how that performance has changed over previous PISA assessments and examines how opportunities to learn are associated with mathematics performance. It also provides an overview of student performance in reading and science, and describes the evolution of performance in these subjects over previous PISA assessments.

Volume II, *Excellence through Equity: Giving Every Student the Chance to Succeed*, defines and measures equity in education and analyses how equity in education has evolved across countries between PISA 2003 and 2012. The volume examines the relationship between student performance and socio-economic status, and describes how other individual student characteristics, such as immigrant background and family structure, and school characteristics, such as school location, are associated with socio-economic status and performance. The volume also reveals differences in how equitably countries allocate resources and opportunities to learn to schools with different socio-economic profiles. Case studies, examining the policy reforms adopted by countries that have improved in PISA, are highlighted throughout the volume.

Volume III, *Ready to Learn: Student Engagement, Drive and Self-Beliefs*, explores students' engagement with and at school, their drive and motivation to succeed, and the beliefs they hold about themselves as mathematics learners. The volume identifies the students who are at particular risk of having low levels of engagement in, and holding negative dispositions towards, school in general and mathematics in particular, and how engagement, drive, motivation and self-beliefs are related to mathematics performance. The volume identifies the roles schools can play in shaping the well-being of students and the role parents can play in promoting their children's engagement with and dispositions

towards learning. Changes in students' engagement, drive, motivation and self-beliefs between 2003 and 2012, and how those dispositions have changed during the period among particular subgroups of students, notably socio-economically advantaged and disadvantaged students, boys and girls, and students at different levels of mathematics proficiency, are examined when comparable data is available. Throughout the volume, case studies examine in greater detail the policy reforms adopted by countries that have improved in PISA.

Volume IV, *What Makes Schools Successful? Resources, Policies and Practices*, examines how student performance is associated with various characteristics of individual schools and of concerned school systems. It discusses how 15-year-old students are selected and grouped into different schools, programs, and education levels, and how human, financial, educational and time resources are allocated to different schools. The volume also examines how school systems balance autonomy with collaboration, and how the learning environment in school shapes student performance. Trends in these variables between 2003 and 2012 are examined when comparable data is available, and case studies, examining the policy reforms adopted by countries that have improved in PISA, are highlighted throughout the volume.

Volume V, *Skills for Life: Student Performance in Problem Solving*, presents student performance in the PISA 2012 assessment of problem solving, which measures students' capacity to respond to non-routine situations in order to achieve their potential as constructive and reflective citizens. It provides the rationale for assessing problem-solving skills and describes performance within and across countries. In addition, the volume highlights the relative strengths and weaknesses of each school system and examines how they are related to individual student characteristics, such as gender, immigrant background and socio-economic status. The volume also explores the role of education in fostering problem-solving skills.

Volume VI, *Students and Money: Financial Literacy Skills for the 21st Century*, examines 15-year-old students' performance in financial literacy in the 18 countries and economies that participated in this optional assessment. It also discusses the relationship of financial literacy to students' and their families' background and to students' mathematics and reading skills. The volume also explores students' access to money and their experience with financial matters. In addition, it provides an overview of the current status of financial education in schools and highlights relevant case studies.

The frameworks for assessing mathematics, reading and science in 2012 are described in *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy* (OECD, 2013).

Notes

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".

2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Reference

OECD (2013a), *OECD Skills Outlook 2013: First Results from the Survey of Adult Skills*, OECD Publishing, Paris.
http://dx.doi/org/10.1787/9789264204256-en

2

Viewing the United States School System through the Prism of PISA

This chapter compares the United States with education systems that have performed well or are rapidly improving on PISA and other international benchmarks. It provides a backdrop for the subsequent chapters, which examine the performance of U.S. students in finer detail, including in relation to the Common Core State Standards.

This chapter compares the United States with education systems that have performed well or are rapidly improving on PISA and other international benchmarks. It provides a backdrop for the subsequent chapters, which examine the performance of United States' students in finer detail, including in relation to the Common Core Standards. The chapter draws out some lessons to be learned for the United States from both the comparative data and the countries portrayed in this volume.

Since the focus of the PISA 2012 assessment was on mathematics, this chapter examines the results for mathematics in greater detail than those for reading and science. Unless noted otherwise, references to tables and figures refer to the PISA 2012 report.

LEARNING OUTCOMES

The United States remains in the middle of the rankings

Among the 34 OECD countries, the United States performed below average in mathematics (rank 26[1]) and around the average in reading (rank 17[2]) and science (rank 21[3]) in the 2012 PISA assessment of 15-year-olds (Table 2.1). Figures 2.12, 2.13 and 2.14 at the end of this chapter show the relative standing of the United States compared with OECD and other countries.

■ Table 2.1. ■
United States' mean scores in mathematics, reading and science

	PISA 2000	PISA 2003	PISA 2006	PISA 2009	PISA 2012
	Mean score	Mean score	Mean score	Mean score	Mean score
Mathematics		483	474	487	481
Reading	504	495		500	498
Science			489	502	497

Source: OECD, 2013a.

There is, of course, significant performance variability within the United States, including between individual states. Unlike other federal nations, the United States did not measure the performance of all states individually, but students in three states – Florida, Connecticut and Massachusetts – were oversampled so as to give state-level results for these states. In mathematics, Massachusetts scored highest of the three, with 514 points (comparable with the performance of Germany), followed by Connecticut with 506 points (comparable with the performance of Austria) and then Florida with 467 points (comparable with the performance of Israel). This ordering of the three states was repeated both for reading and science performance.

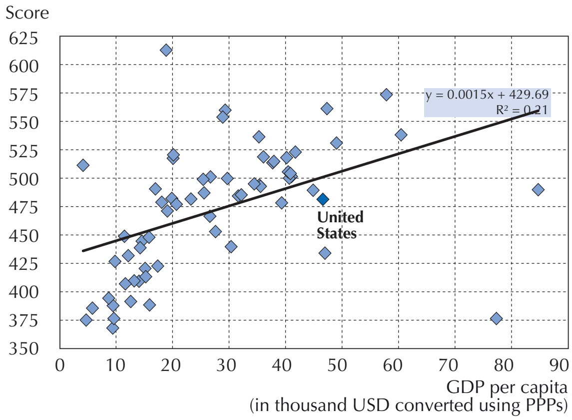
Performance varies even more between schools and social contexts. For example, despite the fact that the relationship between socio-economic background and learning outcomes is stronger in the United States than in most of the top-performing systems, around half of the students in disadvantaged schools have average or better achievement in mathematics.[4]

Based on annualized changes in performance, student performance in mathematics in the United States has shown no significant change since 2003, the first year from which mathematics trends can be measured. Similarly, there has been no significant change in reading performance since 2000 and none in science since 2006.

Average performance needs to be seen against a range of socio-economic background indicators, most of which give the United States a significant advantage compared with other industrialized countries (see Box 2.1 and OECD, 2013a: Table I.2.27).

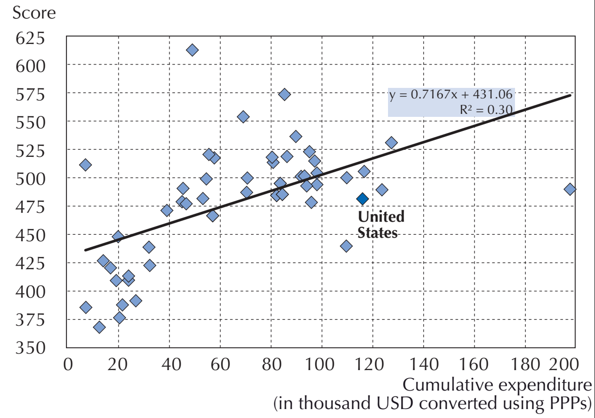
Box 2.1 A context for interpreting the performance of countries

■ Figure 2.1a ■
Mathematics performance and Gross Domestic Product



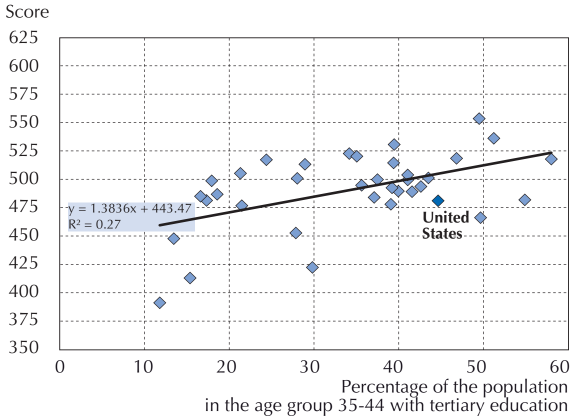
Source: OECD, PISA 2012 Database, Table I.2.27.

■ Figure 2.1b ■
Mathematics performance and spending on education



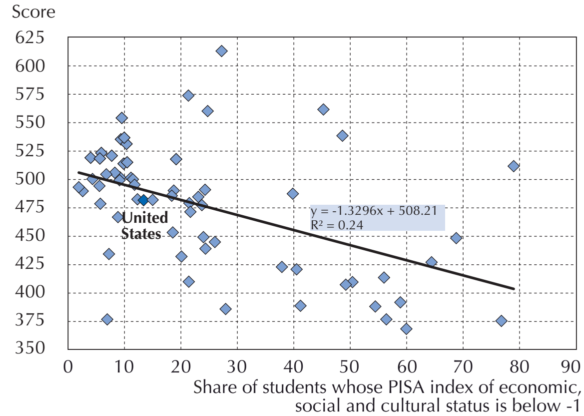
Source: OECD, PISA 2012 Database, Table I.2.27.

■ Figure 2.1c ■
Mathematics performance and parents' education



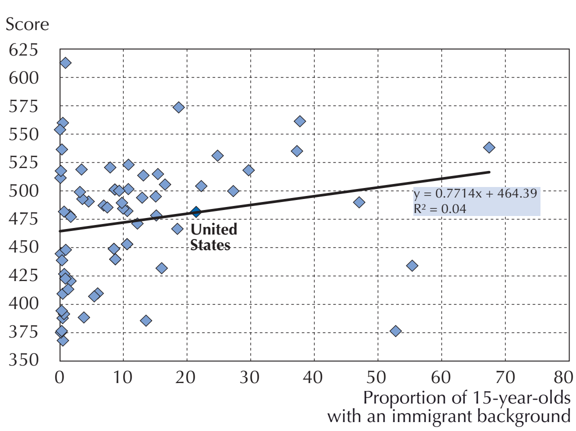
Source: OECD, PISA 2012 Database, Table I.2.27.

■ Figure 2.1d ■
Mathematics performance and share of socio-economically disadvantaged students



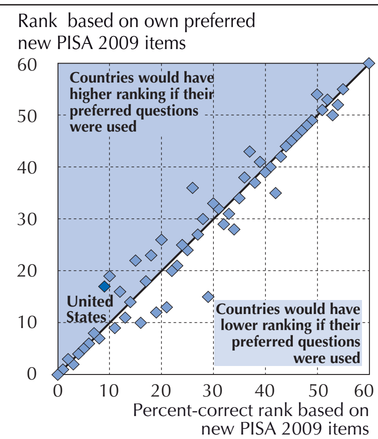
Source: OECD, PISA 2012 Database, Table I.2.27.

■ Figure 2.1e ■
Mathematics performance and proportion of students from an immigrant background



Source: OECD, PISA 2012 Database, Table I.2.27.

■ Figure 2.1f ■
Equivalence of the PISA assessment across cultures and languages



Source: OECD, PISA 2012 Database, Table I.2.28.

Comparing mathematics performance, and educational performance more generally, poses numerous challenges. When teachers give a mathematics test in a classroom, students with varying abilities, attitudes and social backgrounds are required to respond to the same set of tasks. When educators compare the performance of schools, the same test is used across schools that may differ significantly in the structure and sequencing of their curricula, in the pedagogical emphases and instructional methods applied, and in the demographic and social contexts of their student populations. Comparing performance across countries adds more layers of complexity, because students are given tests in different languages, and because the social, economic and cultural context of the countries being compared are often very different. However, even though students within a country may learn in different contexts depending on their home background and the school that they attend, their performance is measured against common standards, since, when they become adults, they will all face common challenges and have to compete for the same jobs. Similarly, in a global economy, the benchmark for success in education is no longer improvement against national standards alone, but increasingly in relation to the best-performing education systems internationally. As difficult as international comparisons are, they are important for educators, and PISA goes to considerable lengths to ensure that such comparisons are valid and fair.

This box discusses countries' mathematics performance in the context of important economic, demographic and social factors that can influence assessment results. It provides a framework for interpreting the results that are presented later in the chapter.

- ***The wealth of the United States means it can spend more on education.*** As shown in the PISA 2012 results (OECD, 2013b), family wealth influences the educational performance of children. Similarly, the relative prosperity of some countries allows them to spend more on education, while other countries find themselves constrained by a lower national income. In fact, 12% of the variation between OECD countries' mean scores can be predicted on the basis of per capita gross domestic product (GDP). The United States, which ranks 3rd after Luxembourg and Switzerland in terms of per capita GDP, has a substantial economic advantage over many other OECD countries because of the amount of money it has available to spend on education (Figure 2.1a and OECD 2013a, Table I.1.27).
- ***Only Austria, Luxembourg, Norway and Switzerland spend more per student.*** While per capita GDP reflects the potential resources available for education in each country, it does not directly measure the financial resources actually invested in education. However, a comparison of countries' actual spending per student, on average, from the age of 6 up to the age of 15 also puts the United States at an advantage, since only Austria, Luxembourg, Norway and Switzerland spend more, on average, on school education per student. Across OECD countries, expenditure per student explains 17% of the variation in mean PISA performance between countries. Deviations from the trend line, however, suggest that moderate spending per student cannot automatically be equated with poor performance by education systems. For example, the Slovak Republic, which spends around USD 53 000 per student, performs at the same level as the United States, which spends over USD 115 000 per student.[5] Similarly, Korea, the highest-performing OECD country in mathematics, spends well below the average on each student (Figure 2,1b and OECD 2013a, Table I.2.27).
- ***Money needs to be directed where it can make the most difference.*** It is not just the volume of resources that matters but also how countries invest these, and how well they succeed in directing the money where it can make the most difference. In some countries, students in socio-economically disadvantaged schools have to cope with less favourable student-teacher ratios and have less well-qualified teachers than in socio-economically advantaged schools. In the United States, however, there is little difference in the student-teacher ratios between advantaged and disadvantaged schools. Similarly, there is no difference between advantaged and disadvantaged schools in terms of the proportion of teachers who have a university-level qualification. The United States spends a far lower proportion than the average OECD country on the salaries of high-school teachers. At the same time, high-school teachers in the United States teach far more hours, which reduces costs, but smaller class sizes are driving costs upward (OECD, 2013e: Table B7.4a). By contrast, Japan and Korea pay their teachers comparatively well and provide them with ample time for work other than teaching, which drives costs upward, while paying for this with comparatively large class sizes. Finland puts emphasis on non-salary aspects of the working conditions of high-school teachers and also pays for the costs with comparatively large class size. Finally, the OECD indicators also show that the United States spends 11.4% of its resources for schools on capital outlays, a figure that is notably higher than the OECD average of 8.7% (OECD 2013e, Table B6.2b).

- ***Parents in the United States are better educated than those in most other countries.*** Given the close relationship between a student's performance and his or her parents' level of education, it is also important to bear in mind the educational attainment of adult populations when comparing the performance of OECD countries. Countries with more highly educated adults are at an advantage over countries where parents have less education. Figure 1.2c shows the percentage of 35-44 year-olds who have attained tertiary education. This group corresponds roughly to the age group of parents of the 15-year-olds assessed in PISA. Parents' level of education explains 27% of the variation in mean performance between countries and economies (23% of the variation among OECD countries). The United States ranks sixth highest among OECD countries on this measure.
- ***The share of students from disadvantaged backgrounds in the United States is about average.*** Differences in the socio-economic background of student populations pose another major challenge for teachers and education systems. As Volume II of the 2012 PISA results have shown (OECD, 2013b), teachers instructing socio-economically disadvantaged children are likely to face greater challenges than those teaching students from more advantaged backgrounds. Similarly, countries with larger proportions of disadvantaged children face greater challenges than countries with smaller proportions of these students. Figure 1.2d shows the proportion of students at the lower end of an international scale of the economic, social and cultural status of students, which is described in detail in Volume II, and how this relates to mathematics performance. The relationship explains 24% of the performance variation among countries (46% of the variation among OECD countries). A comparison of the socio-economic background of the most disadvantaged quarter of students puts the United States around the OECD average while the socio-economic background of the student population as a whole ranks clearly above the OECD average.[6] In other words, while the socio-economic context of students in the United States overall is above that of a typical OECD country, the proportion of students from disadvantaged backgrounds is similar to that of OECD countries in general. The greater socio-economic variability in the United States thus does not result from a disproportionate share of students from poor families, but rather from an above-average share of students from socio-economically advantaged backgrounds.
- ***Among OECD countries, the United States has the 6th largest proportion of students with an immigrant background.*** Integrating students with an immigrant background is part of the socio-economic challenge. The PISA performance levels of students who immigrated to the country in which they were assessed can only be partially attributed to the education system of their host country. The United States has the 6th highest share of students with an immigrant background among OECD countries, at 21.4%. However, the share of students with an immigrant background explains just 4% of the performance variation between countries. Despite having large proportions of immigrant students, some countries, like Canada, perform above the OECD average. Eight OECD countries have between 15% and 30% of students with an immigrant background, including the United States. Of these, four show a smaller PISA performance gap for immigrants than the United States, while three show a larger performance gap (Figure 1.2e and OECD 2013b, Table II.3.4a).

The data in Box 2.1 show that countries vary in their demographic, social and economic contexts. These differences need to be taken into account when interpreting differences in student performance. At the same time, the future economic and social prospects of both individuals and countries depend on the results they actually achieve, not on the performance they might have achieved under different social and economic conditions. That is why the results actually achieved by students, schools and countries are the focus of the subsequent analysis in this chapter.

Even after accounting for the demographic, economic and social contexts of education systems, the question remains: to what extent is an international test meaningful when differences in languages and cultures lead to very different ways in which subjects such as language, mathematics or science are taught and learned across countries? It is inevitable that not all tasks on the PISA assessments are equally appropriate in different cultural contexts and equally relevant in different curricular and instructional contexts. To gauge this, PISA asked every country to identify those tasks from the PISA tests that it considered most appropriate for an international test. Countries were advised to give an on-balance rating for each task with regard to its relevance to "preparedness for life", authenticity and interest for 15-year-olds. Tasks given a high rating by each country are referred to as that country's most preferred questions for PISA. PISA then scored every country on its own most preferred questions and compared the resulting performance with the performance on the entire set of PISA tasks. For the United States, its relative standing remains the same, irrespective of whether all PISA items or the items "preferred" by the United States are used as a basis for comparisons.

■ Figure 2.2 ■

Summary descriptions for the six levels of proficiency in mathematics

Level	Lower score limit	Percentage of students able to perform tasks at each level or above (OECD average)	What students can typically do
6	669	3.3%	At Level 6, students can conceptualize, generalize and utilize information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations. Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situation.
5	544	12.6%	At Level 5, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They begin to reflect on their work and can formulate and communicate their interpretations and reasoning.
4	545	30.8%	At Level 4, students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilize their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
3	482	54.5%	At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning.
2	420	77.0%	At Level 2, students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers. They are capable of making literal interpretations of the results.
1	358	92.0%	At Level 1, students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.

Relative shares of students "at risk"

Just over one-quarter (26%) of 15-year-olds in the United States do not reach the PISA baseline Level 2 of mathematics proficiency. This percentage is higher that the OECD average of 23% and has remained unchanged since 2003. Excluding students with an immigrant background reduces the percentage of poorly performing students slightly to 16%. By contrast, in Canada, Hong Kong-China, Korea, Shanghai-China and Singapore, the proportion of poor performers is around 10% or less (OECD 2013a, Figure I.2.22).

Level 2 on the PISA mathematics scale can be considered a baseline level of proficiency at which students begin to demonstrate the skills that will enable them to participate effectively and productively in life. Students proficient at Level 2 can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant

information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers. They are capable of making literal interpretations of the results.

Results from longitudinal studies in Australia, Canada, Denmark and Switzerland show that students who do not reach Level 2 often face severe disadvantages in their transition into higher education and the labour force in subsequent years. The proportion of students who perform below this baseline proficiency level thus indicates how well countries are performing at providing their populations with a minimum level of competence (OECD, 2012).

For example, the follow-up of students who were assessed by PISA in 2000 as part of the Canadian Youth in Transition Survey shows that students scoring below Level 2 face a disproportionately higher risk of poor post-secondary participation or low labour-market outcomes at age 19, and even more so at age 21, the latest age for which data are currently available. The odds of Canadian students who had reached PISA Level 5 in reading at age 15 achieving a successful transition to post-secondary education by age 21 were 20 times higher than for those who had not achieved the baseline Level 2, even after adjustments for socio-economic differences were made (OECD, 2010a).[7] Similarly, over 60% of the Canadian students who performed below Level 2 in 2000 had not gone on to any post-school education by the age of 21; by contrast, more than half of the students (55%) who had reached Level 2 as their highest level were at college or university.

In reading, the proportion of students in the United States below Level 2 on the PISA reading scale is 16.6% against an OECD average of 18.0%, representing a slight improvement over 2000 when the figure was 17.9% (OECD, 2013a: Table I.4.1b). Students proficient at Level 2 in reading are capable of very basic tasks such as locating information that meets several conditions, making comparisons or contrasts around a single feature, working out what a well-defined part of a text means even when the information is not prominent, and making connections between the text and personal experience.

In science, 18.1% of students in the United States did not reach Level 2 on the PISA science scale, around the OECD average. This shows an improvement over 2006, when the proportion was 24.4% (OECD, 2013a: Table I.5.1b). To reach Level 2 requires competencies such as identifying key features of a scientific investigation, recalling single scientific concepts and information relating to a situation, and using results of a scientific experiment represented in a data table in support of a personal decision. In contrast, students who do not reach Level 2 in science often confuse key features of an investigation, apply incorrect scientific information and mix personal beliefs with scientific facts in support of a decision.

Relative shares of top-performing students

At the other end of the performance scale, the United States has a below-average share of top performers in mathematics. It does slightly better in reading and science where the proportion of top performers is around the OECD average (OECD 2013a, Figures I.2.23, I.4.11 and I.5.11).

Only 2% of students in the United States reach the highest level (Level 6) of performance in mathematics, compared with an OECD average of 3%, and figures of up to 31% in Shanghai-China (OECD 2013a, Table I.2.1a).

Students proficient at Level 6 of the PISA mathematics assessment are able to successfully complete the most difficult PISA items. At Level 6, students can conceptualize, generalize and use information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and move flexibly among them. Students at this level are capable of advanced mathematical thinking and reasoning. They can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations. Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations and arguments, and can explain why they were applied to the original situation.

At the next highest level, Level 5 on the PISA mathematics scale, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations, and insights pertaining to these situations. They begin to reflect on their work and can formulate and communicate their interpretations and reasoning.

Some 8.8% of students in the United States reach the PISA mathematics Level 5, compared with 12.6% on average across OECD countries. In Shanghai-China, over half of the students reach Level 5, while in Hong Kong-China, Korea, Singapore and Chinese Taipei, 30% or more do, and in Japan, Liechtenstein, Macao-China and Switzerland over 20% do.

In reading, students proficient at the top level on the PISA reading scale, Level 6, are capable of conducting fine-grained analysis of texts, which requires detailed comprehension of both explicit information and unstated implications. They are capable of reflecting on and evaluating what they read at a more general level. They can overcome preconceptions in the face of new information, even when that information is contrary to expectations. They are capable of recognizing what is provided in a text, both conspicuously and more subtly, while at the same time being able to apply a critical perspective to it, drawing on sophisticated understandings from beyond the text. This combination of a capacity to absorb the new and to evaluate it is greatly valued in knowledge economies, which depend on innovation and nuanced decision making that draws on all the available evidence. At 1.0%, the United States has an average share of the highest-performing readers, when compared with the share among OECD countries. However, in Singapore the share is 5% and in Japan, New Zealand and Shanghai-China it is 3% or more.

At the next highest level, Level 5 on the PISA reading literacy scale, students can still handle texts that are unfamiliar in either form or content. They can find information in such texts, demonstrate detailed understanding and infer which information is relevant to the task. Using such texts, they are also able to evaluate critically and build hypotheses, draw on specialized knowledge and accommodate concepts that may be contrary to expectations. In the United States, 8% of students perform at Level 5 or above, an average share. However, in Shanghai-China (25.1%), Singapore (21.2%), Japan (18.5%) and Hong-Kong China (16.8%) the corresponding percentages are higher.

Students proficient at Level 6 in science can consistently identify, explain and apply scientific knowledge and knowledge about science in a variety of complex life situations. They can link different information sources and explanations and use evidence from those sources to justify decisions. They clearly and consistently demonstrate advanced scientific thinking and reasoning, and they use their scientific understanding to solve unfamiliar scientific and technological situations. Students at this level can use scientific knowledge and develop arguments in support of recommendations and decisions that center on personal, social or global situations. In the United States, 1% of students reaches Level 6 in science, which corresponds to the OECD average. In Singapore, the percentage is 5.8%, in Shanghai-China 4.2%, in Japan 3.4% and in Finland 3.2%.

Students proficient at the PISA science Level 5 can identify the scientific components of many complex life situations, apply both scientific concepts and knowledge about science to these situations, and can compare, select and evaluate appropriate scientific evidence for responding to life situations. Students at this level can use well-developed inquiry abilities, link knowledge appropriately and bring critical insights to situations. They can construct explanations based on evidence and arguments that emerge from their critical analysis. In the United States, 9% of students reach this level, which again corresponds to the OECD average. In Shanghai-China, 27.2% of students do, while in Singapore the percentage is 22.7%, in Japan 18.2%, in Finland 17.1% and in Hong Kong-China 16.7%.

EQUITY IN THE DISTRIBUTION OF LEARNING OPPORTUNITIES

PISA explores equity in education from three perspectives. First, it examines differences in the distribution of learning outcomes of students and schools. Second, it studies the extent to which students and schools of different socio-economic backgrounds have access to similar educational resources, both in terms of quantity and quality. Third, it looks at the impact of students' family background and school location on learning outcomes. The first perspective was discussed in the previous section; the last two are discussed below.

Learning opportunities

Previous research has shown a relationship between students' exposure to subject content in school, what is known as "opportunity to learn", and student performance (see OECD,2013a references). Building on previous measures of opportunity to learn, the PISA 2012 assessment included questions to students on the mathematics theories, concepts and content to which they have been exposed in school, and the amount of class time they spent studying this content.

The results show that students in the high-performing East Asian countries and economies – Shanghai-China, Singapore, Hong Kong-China, Chinese Taipei, Korea, Macao-China and Japan – are more frequently exposed to formal mathematics than students in the remaining PISA-participating countries and economies on average. Students in the United States report relatively high exposure to both formal mathematics - close to the level of the East Asian countries and economies, in fact – and also relatively high exposure to applied mathematics (OECD 2013a, Figure I.3.17).

The results also show that exposure to more advanced mathematics content, such as algebra and geometry, appears to be related to high performance on the PISA mathematics assessment, even if the causal nature of this relationship cannot be established. At the same time, strong mathematics performance in PISA is not only related to opportunities to learn formal mathematics, such as solving a quadratic equation, using complex numbers, or calculating the volume of a box, but also to opportunities to learn applied mathematics (using mathematics in a real-world context).

Equity in access to resources

A first potential source of inequities in learning opportunities lies in the distribution of resources across students and schools. In a school system characterized by an equitable distribution of educational resources, the quality or quantity of school resources would not be related to a school's average socio-economic background, as all schools would enjoy similar resources. A positive relationship between the socio-economic background of students and schools and the quantity or quality of resources signals that more advantaged schools enjoy more or better resources. A negative relationship implies that more or better resources are devoted to disadvantaged schools. No relationship implies that resources are distributed similarly among schools, whatever the socio-economic background of the students.

Examination of the results from PISA 2012 shows that for students attending disadvantaged schools, quantity of resources does not necessarily translate into quality of resources. In general across countries, more disadvantaged students attend schools with lower student-staff ratios, but more advantaged students attend schools that have a higher proportion of full-time teachers with a university degree. In the United States, however, there is no significant difference between advantaged and disadvantaged schools in terms of student-teacher ratios or the proportion of mathematics teachers qualified to university level.

In around half of OECD countries, the student-teacher ratio is more favourable in disadvantaged schools compared with advantaged schools – in other words, disadvantaged schools tend to have more teachers per student. This is the case in Belgium, Canada, Denmark, Estonia, Finland, Germany, Greece, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Norway, Poland, Portugal, Slovenia, Spain and Sweden. This suggests that these countries use the student-teacher ratio to moderate disadvantage. Among OECD countries, only Turkey has a significantly favourable ratio of teachers to students in advantaged schools.

In just under half of OECD countries, however, more advantaged students enjoy a higher proportion of teachers with university degrees. For example, in the Netherlands the proportion of qualified teachers in socio-economically advantaged schools is more than 40% higher than the proportion of qualified teachers in disadvantaged schools (31% versus 14%). All of this suggests that ensuring an equitable distribution of resources is still a major challenge for many countries, if not in terms of the quantity of resources, then in terms of their quality.

■ Figure 2.3 ■

Summary of PISA measures of equity in educational resources

Disadvantaged schools are more likely to have more or better resources; correlation is stronger than 0.25
Advantaged schools are more likely to have more or better resources; correlation is stronger than 0.25

	Simple correlation between the school mean socio-economic profile and:						
	Student-teacher ratio[1]	Composition and qualifications of mathematics teaching staff (proportion of teachers with university-level qualifications)	Student-related factors affecting school climate	Proportion of students who leave school without a certificate	Parental pressure to achieve	Attendance at after-school lessons	Hours spent on homework or other study set by teachers
OECD average	0.16	0.14	0.30	-0.28	0.31	0.10	0.18
OECD							
Australia	**-0.05**	**0.02**	**0.52**	**-0.31**	0.36	**0.14**	0.25
Austria	**-0.11**	**0.60**	0.23	-0.22	0.25	0.12	**0.23**
Belgium	**0.59**	**0.61**	**0.56**	**-0.36**	0.30	**0.17**	0.31
Canada	0.20	**0.02**	0.36	-0.31	**0.41**	0.10	**0.18**
Chile	**-0.03**	0.19	**0.45**	**-0.34**	**0.44**	0.08	0.16
Czech Republic	0.05	**0.28**	0.31	-0.18	0.28	**0.02**	0.14
Denmark	0.20	0.09	0.35	-0.30	0.35	**0.00**	0.05
Estonia	**0.45**	c	**0.09**	**-0.12**	**0.13**	**0.02**	**0.04**
Finland	**0.36**	0.01	**0.01**	**0.02**	**0.14**	**0.05**	**0.05**
France	w	w	w	w	w	0.13	**0.29**
Germany	0.19	m	0.29	-0.18	**0.13**	0.08	**0.14**
Greece	0.18	0.19	**0.14**	**-0.37**	0.35	**0.21**	0.20
Hungary	**-0.04**	0.16	**0.47**	**-0.43**	**0.49**	**0.20**	0.32
Iceland	**0.42**	**0.18**	**-0.01**	**-0.07**	**0.24**	**0.05**	**0.11**
Ireland	**0.32**	-0.08	**0.42**	-0.33	**0.56**	0.10	**0.15**
Israel	**-0.03**	0.21	**0.14**	-0.20	0.37	**-0.06**	0.07
Italy	**0.40**	**0.30**	**0.41**	**-0.35**	0.30	**0.24**	**0.38**
Japan	**0.30**	0.18	0.34	**-0.39**	**0.44**	**0.31**	**0.33**
Korea	0.27	**0.02**	0.25	-0.24	**0.42**	**0.36**	**0.28**
Luxembourg	0.17	**0.46**	**0.47**	**-0.38**	**-0.06**	**0.06**	**0.16**
Mexico	**0.02**	**0.01**	**0.12**	**-0.02**	**0.10**	0.09	0.16
Netherlands	**0.43**	**0.51**	0.21	-0.34	0.39	0.12	0.22
New Zealand	0.15	0.21	**0.53**	**-0.80**	**0.44**	0.14	0.24
Norway	0.27	c	0.28	c	**0.47**	0.09	**0.12**
Poland	0.07	**-0.07**	**0.04**	**-0.05**	**0.07**	**0.01**	**0.03**
Portugal	**0.41**	**-0.15**	0.17	**0.08**	0.38	0.12	**0.17**
Slovak Republic	0.04	**-0.15**	0.25	-0.28	0.30	**-0.01**	0.16
Slovenia	**0.25**	**0.43**	0.27	-0.23	0.27	**0.04**	0.16
Spain	0.17	**-0.04**	**0.45**	-0.31	0.27	**0.04**	0.08
Sweden	0.26	0.12	**0.43**	**-0.49**	0.40	0.11	**0.17**
Switzerland	**-0.07**	0.18	**0.08**	c	**-0.10**	0.06	0.12
Turkey	**-0.37**	0.04	0.31	-0.19	0.21	**0.05**	**0.04**
United Kingdom	**-0.18**	**0.00**	0.35	-0.29	**0.48**	0.16	**0.31**
United States	**0.02**	-0.02	**0.42**	-0.31	**0.47**	0.14	**0.25**
Partners							
Albania	m	m	m	m	m	m	m
Argentina	**0.05**	0.17	0.33	-0.24	0.15	**0.04**	0.10
Brazil	**-0.21**	**-0.01**	**0.38**	-0.21	0.31	**0.05**	**0.13**
Bulgaria	**-0.02**	c	0.23	**-0.39**	**0.40**	**0.17**	**0.33**
Chinese Taipei	**-0.01**	0.02	0.36	-0.20	0.29	**0.29**	**0.36**
Colombia	**-0.07**	-0.04	0.25	**-0.06**	**0.07**	0.12	**0.18**
Costa Rica	0.18	0.15	**0.43**	**-0.41**	0.22	0.13	0.22
Croatia	0.22	**0.42**	0.20	-0.22	0.19	0.10	0.24
Hong Kong-China	0.04	0.04	0.21	**0.02**	**-0.07**	**0.20**	**0.14**
Indonesia	**-0.11**	0.20	0.17	-0.19	**-0.06**	**0.14**	0.16
Jordan	**-0.07**	**-0.01**	**0.06**	-0.18	0.19	**-0.03**	0.04
Kazakhstan	0.22	0.21	**-0.04**	**-0.04**	0.20	0.08	**0.13**
Latvia	**0.37**	0.16	**0.01**	-0.14	**0.13**	0.11	0.17
Liechtenstein	**0.50**	**0.46**	**0.45**	c	**-0.56**	0.01	0.12
Lithuania	0.05	0.05	0.24	-0.17	**0.15**	**0.04**	0.16
Macao-China	**-0.05**	**-0.09**	**0.26**	-0.23	**0.16**	**0.15**	0.16
Malaysia	0.08	**-0.10**	**0.41**	-0.23	0.30	0.11	0.18
Montenegro	**0.40**	**0.27**	**0.20**	-0.25	**-0.07**	**0.05**	0.16
Peru	0.20	-0.05	0.29	-0.14	0.18	0.08	0.13
Qatar	**0.07**	**-0.09**	**-0.02**	**-0.06**	**0.19**	**-0.03**	**0.13**
Romania	**-0.19**	**0.24**	0.27	-0.24	**0.06**	0.16	**0.25**
Russian Federation	**0.35**	**0.27**	0.21	**-0.07**	0.26	0.06	**0.09**
Serbia	0.29	0.07	0.24	-0.21	0.31	**0.03**	**0.10**
Shanghai-China	**-0.26**	0.26	0.17	-0.35	0.19	**0.24**	**0.35**
Singapore	0.11	**0.36**	**0.47**	-0.17	**0.38**	0.13	**0.18**
Thailand	0.11	**0.03**	**0.12**	-0.28	0.30	**0.22**	0.24
Tunisia	**0.05**	0.03	**-0.08**	-0.19	0.23	**0.03**	**0.07**
United Arab Emirates	**-0.05**	**-0.05**	**0.11**	-0.22	0.26	**-0.03**	**0.11**
Uruguay	**-0.08**	0.23	**0.54**	-0.35	0.25	0.09	**0.10**
Viet Nam	0.12	0.10	0.20	-0.26	0.24	**0.21**	**0.20**

Note: The data are indicated in bold if within-country correlation is significantly different from the OECD average.
1. Negative correlations indicate more favourable characteristics for advantaged students.
Source: OECD, PISA 2012 Database, Table II.4.6.

Moderating the impact of socio-economic background on learning outcomes

Students who did not surpass the most basic PISA performance level were not a random group. The results show that socio-economic disadvantage has a strong impact on student performance in the United States: 15% of the variation in student performance in the United States is explained by students' socio-economic background, similar to the OECD average. This contrasts with less than 10% in a number of countries and economies, including Finland, Hong Kong-China, Japan and Norway. In other words, in the United States, two students from different socio-economic backgrounds vary much more in their learning outcomes than is normally the case in these other countries. Among OECD countries, the strongest impact of socio-economic background on mathematics performance is found in Chile, France, Hungary and the Slovak Republic where background explains more than 20% of the variation. It is important to emphasize that these countries, including the United States, do not necessarily have a more disadvantaged socio-economic student intake than other countries but that socio-economic differences among students have a particularly strong impact on student learning outcomes.

The comparatively close relationship between the learning outcomes of students in the United States and socio-economic background is not simply explained by a more socio-economically heterogeneous student population or society but, as noted before, mainly because socio-economic disadvantage translates more directly into poor educational performance in the United States than is the case in many other countries.

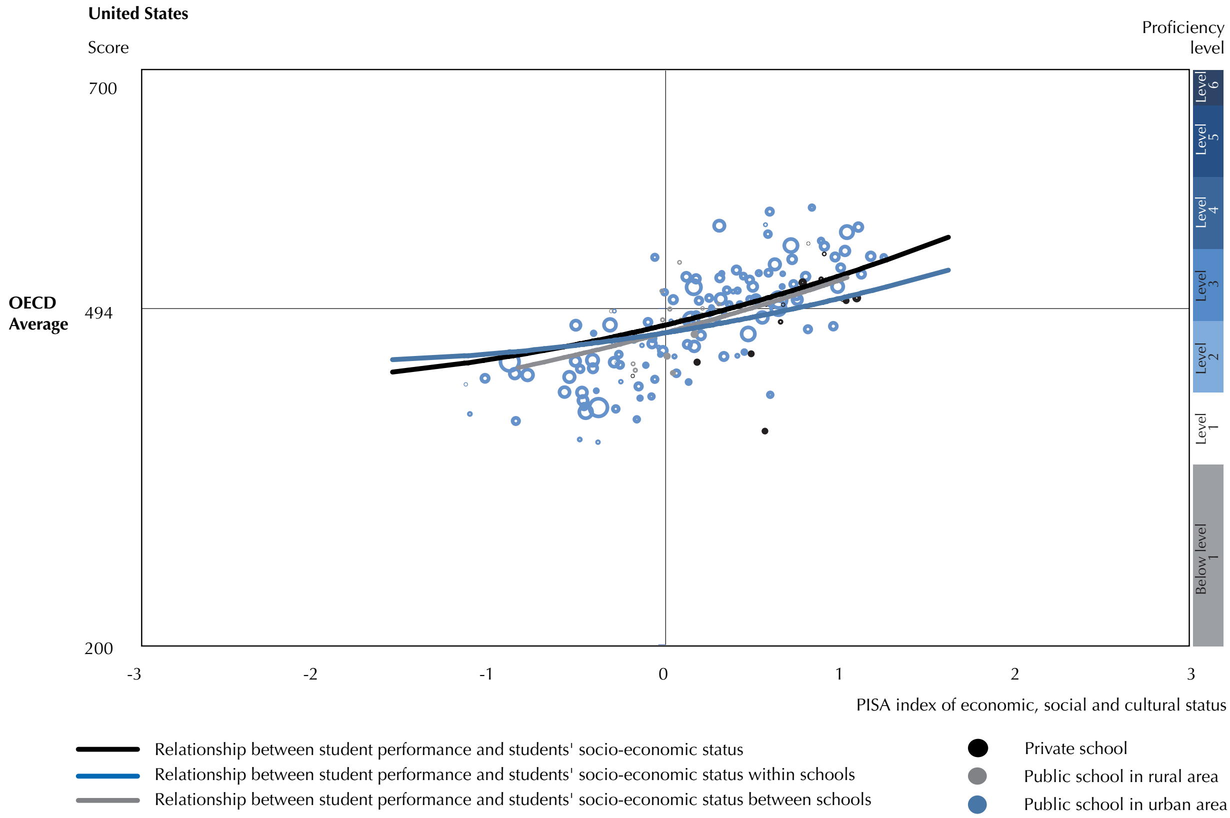
If social inequities in societies were always closely linked to the impact of social disadvantage on learning outcomes, the role of public policy in improving equity in the distribution of learning opportunities would be limited, at least in the short term. However, there is almost no relationship between income inequalities in countries and the impact of socio-economic background on learning outcomes. Some countries succeed, even under difficult conditions, to moderate the impact of socio-economic background on educational success.

The OECD Adult Skills Survey (OECD, 2013f) shows that higher levels of inequality in literacy and numeracy skills are associated with greater inequality in the distribution of income, whatever the causal nature of this relationship. If large proportions of adults have low reading and numeracy skills, the introduction and wider diffusion of productivity-improving technologies and work organization practices can be hampered and that, in turn, will stall improvements in living standards. In other words, today's education is tomorrow's economy.

Even in the United States, the relationship between socio-economic background and learning outcomes is far from deterministic (see Figure 2.4). As noted before, around half of the students in disadvantaged schools have average or better achievement in mathematics.

■ Figure 2.4 ■

Relationship between school performance and schools' socio-economic profile in the United States



It is useful to examine in greater detail four of the aspects of socio-economic background and their relationship to student performance.

- ***Community size***: On average across OECD countries, students in large cities (students attending schools located in cities with over 1 million inhabitants) outperform those in smaller town schools and in rural schools even after taking account of students' socio-economic differences. In the United States, however, these differences are not significant even before adjusting for socio-economic differences among the students. This suggests that the performance challenges for the United States therefore do not just relate to poor students in poor neighbourhoods, but to many students in many neighbourhoods.
- ***Family composition***: While results from PISA show that single-parent families are more prevalent in the United States than on average across OECD countries (20% of 15-year-olds in the United States come from a single-parent family compared with an average of 13%), they also show that 15-year-olds in the United States from single-parent families face a much higher risk of low performance than is the case across OECD countries on average (OECD 2013b, Table II.3.1).
- ***Immigrant students***: Some 22% of 15-year-old students in United States schools have an immigrant background, notably higher than the OECD average of 11% and an increase of 7 percentage points compared with 2003. Immigrant students also tend to be concentrated in certain schools within the system. In the United States, 34% of 15-year-old students are in schools that have more than a quarter of students with an immigrant background. Among OECD countries, only Australia, Canada, Luxembourg, New Zealand and Switzerland show a higher concentration of students with an immigrant background (the OECD average is 15%) (OECD 2013b, Table II.3.9). What PISA data also show is that students in the United States with an immigrant background tend to attend schools with a socio-economically more disadvantaged profile. In fact, in the United States, 40% of students in disadvantaged schools are from immigrant backgrounds, whereas they account for 13% of the student population in advantaged schools (OECD 2013b, Table II.4.5).

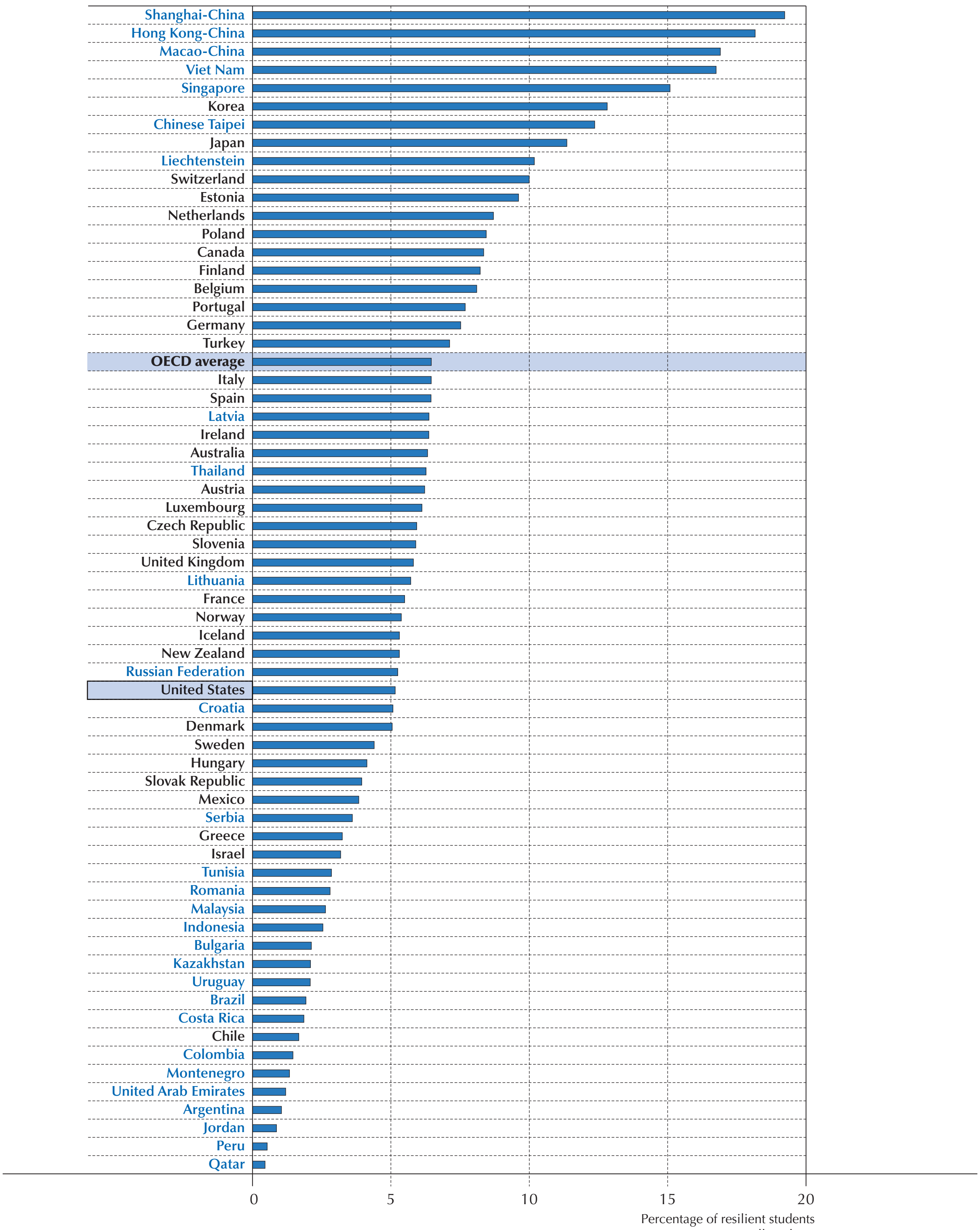
While it might be tempting to attribute countries' poorer performance to the challenges that immigrant inflows pose to the education system, the mathematics performance gap between students with and without an immigrant background is smaller in the United States than the average gap across OECD countries (OECD 2013b, Table II.3.4a). In fact after the socio-economic background of students is accounted for, immigrant students outperform non-immigrant students by 15 PISA score points and this relative performance of immigrant students has improved over time (OECD 2013b, Table II.3.4a). A similar picture is revealed using the language spoken at home as the basis for comparing student groups instead of immigrant background. Indeed, in the United States, there is no performance difference between immigrant students who do not speak the language of assessment at home and non-immigrant students who do. Among the countries that took part in the latest PISA assessment, Australia, Canada, New Zealand and Switzerland have larger immigrant intakes than the United States, but score significantly better overall.

- ***Concentration of disadvantage***: Generally, underperformance among immigrant students can be partly linked to the fact that these students tend to be concentrated in disadvantaged schools. When they move to a new country, people tend to settle in neighbourhoods with other immigrants, often of their own origin and socio-economic status. The PISA 2012 results suggest that, on average across countries, students who attend schools where the proportion of immigrant students is large perform as well as those who attend schools where the proportion of immigrant students is small, after the socio-economic profiles of the students and the school are taken into account. Across OECD countries, students who attend schools where the concentration of immigrants is high (i.e. where more than a quarter of students are immigrants) tend to perform worse than those in schools with no immigrant students. The observed difference between these two groups is 19 score points, but after adjusting for the socio-economic status of the students and schools, the difference is more than halved. The pattern is rather different in the United States, however, where students in schools where there is a high concentration of immigrant students perform, on average, at the same level as students in other schools, before and after adjusting for the socio-economic status of the students and the schools.

In general, the accuracy with which socio-economic background predicts student performance varies considerably across countries. Most of the students who perform poorly in PISA come from challenging socio-economic backgrounds, and yet some of their disadvantaged peers beat the odds and excel in PISA. These "resilient" students show that overcoming socio-economic barriers to achievement is possible.[8] While the prevalence of resilience is not the same across education systems, it is possible to identify substantial numbers of resilient students in practically all OECD countries. In the United States, 5% of students can be considered resilient, in the sense that they are among the 25% most socio-economically disadvantaged students but nevertheless perform much better than would be predicted by their socio-economic background. The average in the OECD is 7% (Figure 2.5). However, in Hong Kong-China, Macao-China, Shanghai-China and Viet Nam, the share of students who excel at school despite their disadvantaged background is about three times higher than it is in the United States.

■ Figure 2.5 ■

Percentage of resilient students



Note: A student is classified as resilient if he or she in the bottom quarter of the *PISA index of economic, social and cultural status* (ESCS) in the country of assessment and performs in the top quarter of students among all countries, after accounting for socio-economic status.
Countries and economies are ranked in descending order of the percentage of resilient students.
Source: OECD, PISA 2012 Database, Table II.2.7a.

THE COST OF THE ACHIEVEMENT GAP

The international achievement gap in education imposes an invisible yet recurring loss on the economy of the United States. Using economic modelling to relate cognitive skills – as measured by PISA and other international instruments – to economic growth shows (with some caveats) that even small improvements in the skills of a nation's labor force can have a large impact on that country's future well-being. A study carried out by the OECD (OECD, 2010b), in collaboration with the Hoover Institute at Stanford University, suggests that a modest goal of having the United States boost its average PISA scores by 25 points over the next 20 years – which corresponds to the performance gains that some countries achieved in the first ten years of PISA alone – could add USD 41 trillion to the United States economy over the lifetime of the generation born in 2010 (as evaluated at the start of reform in terms of the real present value of future improvements in GDP). Bringing the United States up to the average performance of Finland could result in gains on the order of USD 103 trillion. Narrowing the achievement gap by bringing all students to a baseline level of proficiency for the OECD (a PISA score of about 400) could increase the GDP of the United States by USD 72 trillion, according to historical growth relationships (OECD, 2010b).

Although there are uncertainties associated with these estimates, the gains from improved learning outcomes, in terms of current GDP, exceed today's value of the short-run business-cycle management. This is not to say that efforts should not be directed towards mitigating the short-term effects of the economic recession, but that long-term issues should not be neglected.

THE LEARNING ENVIRONMENT IN THE CLASSROOM AND AT SCHOOL

The effects of education policies and practices on student achievement depend heavily on how they translate into increased learning in the classroom. Results from PISA suggest that, across OECD countries, schools and countries where students work in a climate characterized by expectations of high performance and the readiness to invest effort, good teacher-student relations and high teacher morale tend to achieve better results on average across countries, and particularly in some countries.

PISA also shows that the socio-economic background of students and schools and key features of the learning environment are closely inter-related. Both link to performance in important ways, perhaps because students from socio-economically advantaged backgrounds bring with them a higher level of discipline and more positive perceptions of school values, or perhaps because parents' expectations of good classroom discipline are higher, and teacher commitment is stronger, in schools with advantaged socio-economic intakes. Conversely, disadvantaged schools may be under less parental pressure to reinforce effective disciplinary practices or replace absent or unmotivated teachers. In many countries, the effect of parental pressure is particularly closely related to socio-economic background, with little independent effect. However, factors related to the climate within the school, such as discipline and teacher-student relations, are also related to performance independently of any socio-economic and demographic variables. In summary, students perform better in schools with a more positive climate partly because such schools tend to have more students from advantaged backgrounds who generally perform well, partly because the favourable socio-economic characteristics of students reinforce the favourable climate, and partly for reasons unrelated to socio-economic variables.

PISA 2012 results examined disciplinary climate, teacher-student relations, teacher-related factors affecting school climate, student-related factors affecting school climate, students' sense of belonging, teacher morale, and the level of student truancy, including arriving late for school, skipping school and dropping out. The following sections examine some of the factors underlying these analyses in greater detail, as well as the performance of the United States in these areas.

Student truancy

Student truancy tends to be negatively related to a system's overall performance. Among OECD countries, after accounting for per capita GDP, systems with higher percentages of students who arrive late for school tend to have lower scores in mathematics, as do systems with higher percentages of students who skip school. Some 30% of 15-year-old students in the United States reported that they had arrived late for school at least once in the two weeks prior to the PISA test, slightly below the OECD average of 35%. By contrast, around 15% to 19% of students in Hong Kong-China, Liechtenstein, Shanghai-China and Viet Nam had arrived late at least once, and only 9% of students in Japan.

In the United States some 20% of students reported that they had skipped a day of school in the previous two weeks, above the OECD average of 15% and in contrast with Colombia, Hong Kong-China, Iceland, Ireland, Japan, Korea, Liechtenstein, Macao-China, the Netherlands, Shanghai-China, Switzerland and Chinese Taipei, where fewer than 5% of students did so (OECD 2013d, Tables IV.5.1 and IV.5.3).

School climate

Compared with averages for other countries, 15-year-olds in the United States view the relationships between students and teachers relatively positively. All the same, schools in the United States with better average performance tend to have a more positive student-teacher relationships, even after accounting for the socio-economic status and demographic background of students and schools and various other school characteristics.

Disciplinary climate is also consistently related to higher average performance at the school level. In 48 participating countries and economies schools with better average performance tend to have a more positive disciplinary climate, even after accounting for the socio-economic status and demographic background of students and schools and various other school characteristics. However, this relationship does not hold true for schools in the United States

Teacher-student relations

Positive teacher-student relations can help to establish an environment that is conducive to learning. Research finds that students, particularly disadvantaged students, tend to learn more and have fewer disciplinary problems when they feel that their teachers take them seriously. One explanation is that positive teacher-student relations help foster social relationships, create communal learning environments, and promote and strengthen adherence to norms conducive to learning. PISA asked students to indicate whether and to what extent they agree with several statements regarding their relationships with teachers at school, including whether they get along with their teachers, whether teachers are interested in their personal well-being, whether teachers take them seriously, whether teachers are a source of support if they need extra help and whether teachers treat them fairly. Students in the United States reported one of the best teacher-student relations among OECD countries (OECD 2013d, Figure IV.5.3). For example, over 80% of students in the United States agree or strongly agree that their teachers are interested in their well-being, whereas only 59% of students in Japan do so. As in the majority of countries, there is a positive relationship between teacher-student relations and student performance in the United States. For example, the 25% of students in the United States who reported the poorest relationships with their teachers are 1.4 times more likely to be also among the 25% of the poorest performing students. Differences in student-reported teacher interest in their well-being may reflect different student expectations of the level of involvement of their teachers, as well as different roles that teachers assume with respect to their students. In other words, students may disagree with these statements due to a possible mismatch between student expectations and what teachers are actually doing.

According to students' reports, teacher-student relations improved between 2003 and 2012 in all but one country, Tunisia, where they remained stable.

Disciplinary climate

The disciplinary climate in the classroom and school can also affect learning. Classrooms and schools with more disciplinary problems are less conducive to learning since teachers have to spend more time creating an orderly environment before instruction can begin. More interruptions within the classroom disrupt students' engagement and their ability to follow the lessons. PISA asked students to describe the frequency with which interruptions occur in mathematics lessons. The disciplinary climate is measured by how often students do not listen to the teacher during mathematics lessons; whether there is noise and disorder; if the teacher has to wait a long time for students to quiet down; if students cannot work well; and if students do not start working for a long time after the lesson begins. The majority of students in OECD countries enjoy orderly classrooms in their mathematics classes. Some 73% of students report that they never or only in some lessons feel that students do not start working for a long time after the lesson begins; 68% that they never or only in some lessons feel that students do not listen; 68% that noise never or only in some lessons affects learning; 72% that their teacher never or only in some lessons has to wait a long time before students settle down; and 78% of the students attend classrooms where they feel they can work well practically most of the time (OECD 2013d, Figure IV.5.4).

The United States does reasonably well on this measure, but well below Japan, for example, which shows a significantly better disciplinary climate. What is also noteworthy is that there is considerable variation on this measure among students in the United States, and the 25% of students who reported the poorest disciplinary climate are almost twice as likely to be poor performers. This odds ratio is the second highest among all OECD countries, after Israel, against OECD of 1.9 average odds ratio of 1.6 (OECD 2013d, Table IV.5.6).

Between 2003 and 2012, disciplinary climate, as reported by students, improved on average across OECD countries and across 27 individual countries, including the United States.

It is noteworthy that school principals judge disciplinary climate in the United States less positively than students do. The mismatch between these perspectives may indicate differences between what students and school principals perceive to be problems (OECD 2013d, Table IV.5.8).

Teacher-related factors affecting the school climate

To determine the extent to which teacher behavior influences student learning, PISA asked school principals to report the extent to which they perceive learning in their schools to be hindered by such factors as teachers' low expectations of students, poor student-teacher relations, absenteeism among teachers, staff resistance to change, teachers not meeting individual students' needs, teachers being too strict with students and students not being encouraged to achieve their full potential.

The United States performed around the OECD average on most of these measures. Interestingly only 32% of students in the United States are enrolled in schools whose principals reported that students' learning is hindered because teachers have to teach students of differing ability. This is a much lower percentage than across OECD countries, where the average is 55%. Nevertheless, considering that 24% of students in the United States are enrolled in schools whose principals reported that teachers do not meet individual students' needs, these findings do indicate a need to provide teachers with these skills through professional development. In contrast, only 5% of school principals see teachers being too strict with students as a problem, and 10% or less report teacher absenteeism or teachers being late for classes as a problem that hinders learning (Figure 2.6).

Teacher morale

To examine the level of teacher morale in school, school principals were asked to report whether and to what extent they agree with the following statements: the morale of teachers in this school is high, teachers work with enthusiasm, teachers take pride in the school, and teachers value academic achievement. In the United States, 81% of students attend schools whose principals agreed or strongly agreed that the morale of teachers in their schools is high, which is below the OECD average of 91%. For the other indicators of teachers' morale, the United States scores around the OECD average (OECD 2013, Figure IV.5.8).

Inter-relationships among learning-environment indicators at the school level

These indicators of school climate are inter-related. For instance, in virtually all school systems, schools with more negative disciplinary climates tend to have a higher incidence of student truancy (arriving late for school or skipping a day or a class). Also, schools whose principals reported that teachers' behavior negatively affects learning to a great extent also tend to be those whose principals reported that their teachers' morale is low. The United States is one of the countries where this relationship is particularly strong. Similarly, the United States is one of the countries with the strongest correlation between schools with a student population that is predominantly socio-economically disadvantaged and a more negative school disciplinary climate.

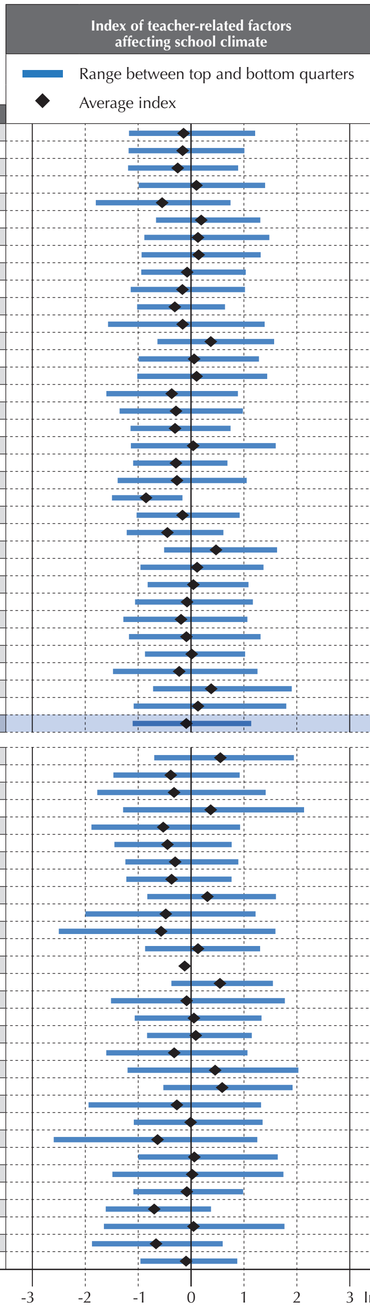
■ Figure 2.6 ■

School principals' views of how teacher behavior affects learning

- **A** Students not being encouraged to achieve their full potential
- **B** Poor teacher-student relations
- **C** Teachers having to teach students of heterogeneous ability levels within the same class
- **D** Teachers having to teach students of diverse ethnic backgrounds (i.e. language, culture) within the same class
- **E** Teachers' low expectations of students
- **F** Teachers not meeting individual students' needs
- **G** Teacher absenteeism
- **H** Staff resisting change
- **I** Teachers being too strict with students
- **J** Teachers being late for classes
- **K** Teachers not being well prepared for classes

Percentage of students in schools whose principals reported that the following phenomena hindered learning "not at all" or "very little"

		A	B	C	D	E	F	G	H	I	J	K	Variability in the index S.D.
OECD	Australia	85	91	66	83	81	65	87	64	94	94	90	0.99
	Austria	86	94	33	65	85	81	80	73	88	85	93	0.88
	Belgium	82	97	42	76	92	84	75	66	86	91	87	0.83
	Canada	90	95	62	80	94	78	91	66	92	99	96	0.97
	Chile	54	88	29	89	66	68	75	57	86	79	73	1.02
	Czech Republic	82	96	40	95	93	96	91	93	91	99	99	0.81
	Denmark	86	97	61	88	91	86	85	84	99	95	97	0.94
	Estonia	72	97	44	89	94	80	92	80	89	96	98	0.89
	Finland	93	95	30	81	97	80	83	78	96	95	96	0.78
	France	78	92	26	82	92	66	91	58	77	98	89	0.88
	Germany	87	98	41	79	92	85	70	75	93	92	96	0.71
	Greece	74	85	31	65	70	80	88	77	87	87	86	1.19
	Hungary	71	93	38	88	96	86	99	95	92	99	100	0.89
	Iceland	86	99	49	77	91	75	85	69	97	93	89	0.92
	Ireland	87	98	67	85	86	82	88	81	89	90	90	0.99
	Israel	75	79	45	83	79	73	74	79	86	83	82	1.02
	Italy	72	74	64	84	79	76	89	47	80	94	87	0.95
	Japan	72	90	28	96	80	74	97	69	81	93	87	0.81
	Korea	80	86	39	98	75	74	99	86	84	96	90	1.14
	Luxembourg	79	92	40	34	96	83	94	80	91	95	99	0.73
	Mexico	61	94	69	92	74	75	83	65	77	80	85	0.99
	Netherlands	35	93	41	85	75	29	60	54	89	84	82	0.53
	New Zealand	91	96	72	81	86	67	93	73	97	97	94	0.79
	Norway	75	90	30	76	82	56	70	74	99	87	94	0.80
	Poland	93	99	22	99	96	90	93	89	97	98	97	0.86
	Portugal	76	97	32	91	83	88	98	82	98	97	94	0.95
	Slovak Republic	79	98	38	85	88	92	92	84	76	99	96	0.76
	Slovenia	84	95	31	74	86	83	87	78	90	95	95	0.92
	Spain	71	94	34	71	78	76	95	68	85	91	93	0.94
	Sweden	79	93	55	70	81	74	79	79	97	97	92	1.02
	Switzerland	89	98	44	56	96	87	95	75	94	98	98	0.77
	Turkey	68	82	39	95	68	46	89	76	93	93	78	1.12
	United Kingdom	93	100	86	95	95	81	84	85	99	99	96	1.05
	United States	89	94	68	76	83	76	91	72	95	99	93	1.16
	OECD average	79	93	45	81	85	76	87	74	90	93	92	0.91
Partners	Albania	87	93	57	90	79	91	94	92	86	97	94	1.06
	Argentina	48	91	51	94	72	73	41	55	83	77	82	0.93
	Brazil	63	81	59	90	61	59	66	64	83	75	72	1.27
	Bulgaria	72	83	62	89	79	79	84	88	91	87	79	1.33
	Colombia	51	85	20	79	72	67	78	49	66	80	83	1.13
	Costa Rica	55	96	39	91	79	67	72	62	87	80	86	0.91
	Croatia	76	88	28	85	73	76	92	72	80	88	81	0.87
	Hong Kong-China	63	95	18	95	70	55	89	82	94	96	93	0.86
	Indonesia	58	99	48	72	94	97	97	98	96	96	97	0.99
	Jordan	72	77	45	71	64	70	57	65	75	75	70	1.28
	Kazakhstan	55	58	38	80	54	61	57	67	60	58	47	1.61
	Latvia	83	93	25	87	86	87	95	91	91	98	94	0.89
	Liechtenstein	100	93	57	48	100	93	88	74	93	100	100	0.66
	Lithuania	97	99	44	98	93	92	99	95	99	100	96	0.76
	Macao-China	62	83	54	93	78	57	84	82	84	87	78	1.31
	Malaysia	93	95	45	66	87	87	87	89	86	89	90	0.98
	Montenegro	71	100	50	91	78	86	94	84	96	92	97	0.79
	Peru	53	92	65	87	81	69	84	65	73	80	77	1.08
	Qatar	92	90	50	73	84	85	89	86	92	93	93	1.33
	Romania	91	91	79	92	91	94	94	93	92	95	96	0.99
	Russian Federation	45	80	44	92	68	64	74	65	76	76	66	1.27
	Serbia	64	92	61	92	71	88	93	75	88	85	80	0.98
	Shanghai-China	51	66	35	78	63	43	65	50	74	69	55	1.52
	Singapore	90	93	52	73	88	77	96	86	94	96	94	1.09
	Chinese Taipei	78	85	46	81	79	69	91	79	84	92	85	1.28
	Thailand	93	97	26	81	87	86	89	89	66	85	86	0.83
	Tunisia	41	76	42	94	59	66	36	61	71	65	88	0.81
	United Arab Emirates	79	84	61	76	80	73	80	76	82	87	83	1.38
	Uruguay	45	86	25	93	63	63	35	66	89	70	70	1.01
	Viet Nam	79	96	22	89	79	80	96	95	77	96	89	0.72



Note: Higher values on the index indicate better school climate.
Source: OECD, PISA 2012 Database, Table IV.5.7.

HOW SCHOOLING IS ORGANIZED

Governance of school systems

Many countries have pursued a shift in public and governmental concern away from merely controlling the resources and content of education and have focused increasingly on outcomes. Successive PISA assessments have revealed the changing distribution of decision-making responsibilities in education. In addition, school systems have made efforts to devolve responsibility to the frontline, encouraging responsiveness to local needs, and strengthening accountability.

Two important organizational features of school systems are the degree to which students and parents can choose schools, and the degree to which schools are considered autonomous entities that make organizational decisions independently of district, regional or national entities. PISA shows that school systems that grant more autonomy to schools to define and elaborate their curricula and assessments tend to perform better than systems that don't grant such autonomy, even after accounting for countries' national income. School systems that provide schools with greater discretion in deciding student-assessment policies, the courses offered, the content of those courses and the textbooks used are also school systems that perform at higher levels in mathematics, reading and science. In contrast, greater responsibility in managing resources appears to be unrelated to a school system's overall performance.

Of course, the United States has a decentralized education system too. Many systems have decentralized decisions concerning the delivery of educational services while keeping tight control over the definition of outcomes, the design of curricula, standards and testing. The United States is different in that it has decentralized both inputs and control over outcomes. That has only just begun to change with the recent introduction and adoption by individual states of common core educational standards. Moreover, while the United States has devolved responsibilities to local authorities or districts, the schools themselves often have less discretion in decision making than is the case in many OECD countries. In this sense, the question for the United States is not just how many charter schools it establishes but how to build the capacity for all schools to exercise responsible autonomy, as happens in most successful systems.

One aspect of accountability is whether schools publicly post their achievement data. Data from PISA 2012 show that in those systems where a larger share of schools post their achievement data there is a positive relationship between school autonomy in resource allocation and student performance, whereas in systems where schools do not, the relationship is the other way around.

In school systems where schools do not post achievement data, a student who attends a school with greater autonomy in defining and elaborating curricula and assessment policies tends to perform seven points lower in mathematics than a student who attends a school with less autonomy in these areas, after students' and schools' socio-economic status and demographic profile are taken into account. In contrast, in a school system where all schools post achievement data publicly, a student who attends a school with greater autonomy scores eight points higher in mathematics than a student who attends a school with less autonomy (OECD 2013d, Figure IV.1.16).

The relationship between school autonomy and performance also appears to be affected by whether there is a culture of collaboration between teachers and principals in managing a school. Figure IV.1.17in OECD, 2013d shows that, in school systems where principals reported less teacher participation in school management (i.e. 1.5 index points lower than the OECD average), even after students' and schools' socio-economic status and demographic profile are taken into account, a student who attends a school with greater autonomy in allocating resources tends to score 16 points lower in mathematics than a student who attends a school with less autonomy. In contrast, in a school system where principals reported more teacher participation in school management (i.e. 1.5 index points higher than the OECD average), a student who attends a school with greater autonomy scores 9 points higher in mathematics than a student who attends a school with less autonomy.

■ Figure 2.7 ■

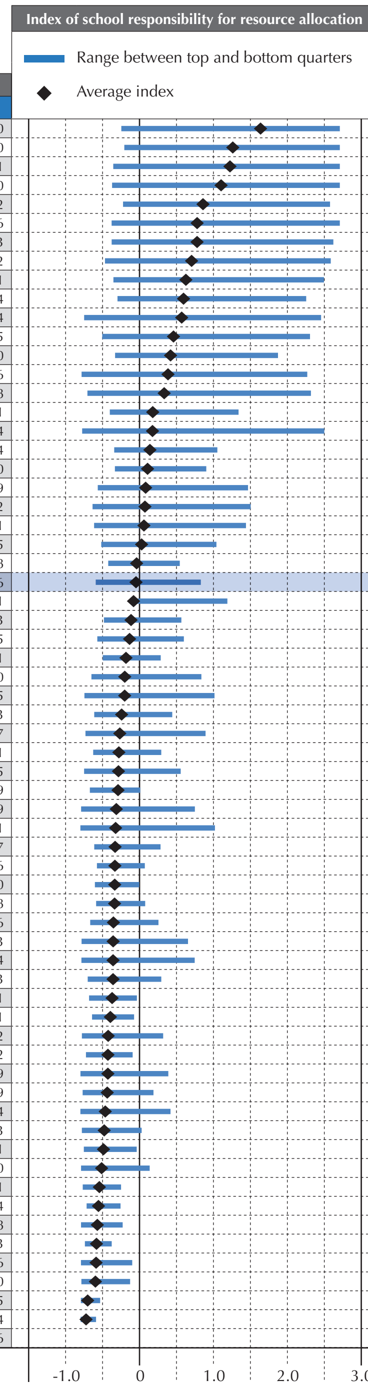
School autonomy over resource allocation

Percentage of students in schools whose principals reported that only "principals and/or teachers", only "regional and/or national education authority", or both "principals and/or teachers" and "regional and/or national education authority", or "school governing board" has/have a considerable responsibility for the following tasks:

- A Selecting teachers for hire
- B Firing teachers
- C Establishing teachers' starting salaries
- D Determining teachers' salaries increases
- E Formulating the school budget
- F Deciding on budget allocations within the school

- 1 Only "principals and/or teachers"
- 2 Both "principals and/or teachers" and "regional and/or national education authority", or "school governing board"
- 3 Only "regional and/or national education authority"

	A			B			C			D			E			F			Variability in the index
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	S.D.
Macao-China	61	37	2	48	49	4	24	72	4	15	81	4	32	68	0	32	68	0	1.2
Netherlands	92	8	0	54	46	0	35	53	12	43	40	17	55	45	0	73	27	0	1.2
Czech Republic	95	5	0	94	6	0	74	18	8	72	22	6	56	36	9	78	21	1	1.4
United Kingdom	52	48	0	23	71	7	31	49	20	19	67	15	12	79	9	49	51	0	1.2
Bulgaria	92	7	1	90	9	1	55	26	18	65	30	5	25	41	33	60	38	2	1.1
Slovak Republic	95	5	0	92	8	0	45	13	42	48	19	33	50	27	23	63	31	6	1.2
Lithuania	82	18	0	84	16	0	38	39	22	33	45	21	15	64	21	30	57	13	1.2
Thailand	32	54	13	39	48	12	22	33	44	54	39	7	32	59	9	45	53	2	1.2
Sweden	90	10	0	44	35	22	40	24	36	52	40	8	45	44	11	86	13	1	1.2
Latvia	92	8	0	88	12	0	29	27	44	33	33	34	34	61	5	31	66	4	1.1
Chile	38	42	20	29	40	31	17	42	41	17	43	40	20	56	24	25	61	14	1.3
Hungary	68	32	0	73	27	0	32	16	52	28	23	49	34	43	23	30	65	5	1.1
Hong Kong-China	52	48	1	14	83	3	7	41	52	4	32	64	35	65	1	39	61	0	1.0
United Arab Emirates	31	29	40	27	35	38	20	37	43	16	43	42	23	51	25	24	50	26	1.2
Indonesia	20	31	48	16	35	48	15	34	51	17	37	46	44	50	6	46	46	8	1.3
Denmark	51	49	0	63	32	5	15	15	70	12	19	69	40	52	8	22	77	1	0.9
Peru	26	28	46	23	20	57	23	4	74	21	5	74	41	40	19	41	45	14	1.4
Estonia	84	16	0	90	10	0	11	14	74	14	30	55	34	54	11	61	35	4	0.8
New Zealand	75	25	0	28	71	0	10	9	82	15	24	61	23	77	0	46	54	0	0.7
United States	59	39	2	37	57	6	3	51	46	2	54	44	6	71	24	30	61	9	0.9
Chinese Taipei	66	24	10	54	36	10	10	14	76	16	14	71	29	34	37	53	35	12	1.0
Australia	63	20	17	39	17	43	8	12	80	9	14	77	38	47	15	62	37	1	1.0
Russian Federation	94	6	0	91	9	0	23	27	50	11	43	46	9	36	55	23	62	15	0.8
Iceland	96	3	1	92	7	1	8	15	77	8	11	81	50	37	13	61	31	8	0.6
OECD average	49	27	24	36	30	34	11	15	73	12	19	69	24	48	28	45	49	6	0.7
Liechtenstein	6	87	6	6	80	13	6	28	65	6	46	47	6	57	37	89	0	11	0.9
Slovenia	93	7	0	63	32	5	7	15	78	10	28	62	13	62	25	26	71	3	0.7
Switzerland	44	54	2	31	58	11	8	15	77	8	23	69	10	63	28	49	46	5	0.6
Norway	82	14	4	55	20	25	5	7	88	2	11	87	54	28	18	75	24	1	0.4
Luxembourg	11	59	30	13	54	33	1	20	79	1	20	79	13	69	18	17	83	0	0.8
Tunisia	32	12	57	31	9	61	28	7	66	28	6	67	33	23	43	40	35	25	0.8
Israel	70	26	3	55	33	13	4	9	86	8	12	80	17	29	54	59	28	13	0.6
Japan	14	19	67	8	25	67	2	29	70	3	32	65	18	28	54	65	27	7	0.8
Finland	41	45	14	23	36	41	7	8	85	7	15	78	31	39	30	87	12	1	0.6
Shanghai-China	34	56	10	19	61	20	2	22	76	5	26	69	15	46	39	26	69	5	0.7
Belgium	57	32	10	32	54	14	0	4	96	0	3	96	18	61	21	32	59	9	0.3
Mexico	26	20	54	16	18	65	6	13	80	6	13	81	31	32	37	44	36	19	0.8
Brazil	18	12	70	17	11	72	11	6	83	11	6	84	15	22	63	14	35	51	1.0
Kazakhstan	81	17	2	76	23	1	19	12	69	4	11	86	8	15	77	11	22	67	0.6
Montenegro	91	9	0	86	7	7	2	0	98	18	1	82	4	41	54	18	66	16	0.5
Croatia	22	77	1	24	73	2	1	1	98	1	2	97	8	68	25	11	80	10	0.3
Poland	80	18	2	76	21	3	7	12	81	5	14	81	4	44	52	25	47	28	0.4
Canada	48	38	14	11	33	56	2	16	82	2	17	82	21	42	37	65	29	6	0.6
Costa Rica	14	7	79	12	7	81	6	9	85	5	8	86	18	63	19	20	78	3	0.9
Colombia	19	3	78	16	5	79	13	2	85	9	3	88	25	40	36	27	68	4	0.9
Singapore	8	36	55	8	37	54	1	8	90	2	29	69	22	59	19	40	57	3	0.7
Qatar	34	50	16	30	50	20	5	46	50	5	44	51	10	56	34	26	53	21	0.4
Serbia	28	69	2	33	64	4	3	4	92	7	12	82	1	35	64	12	78	11	0.3
Spain	22	12	66	19	16	65	2	3	94	3	5	92	30	55	15	30	68	2	0.6
Ireland	19	68	13	1	76	23	0	4	96	0	5	95	11	66	24	36	51	12	0.2
Viet Nam	15	26	60	9	24	67	6	9	84	21	16	63	25	19	56	33	18	49	0.7
Korea	38	13	49	27	14	59	7	3	91	4	3	93	15	33	52	41	50	9	0.6
Uruguay	8	17	76	4	20	76	1	20	79	1	19	81	5	30	65	18	37	44	0.7
Portugal	21	55	24	14	16	69	1	8	91	1	8	91	23	59	18	33	64	3	0.5
Malaysia	7	13	79	4	14	83	1	6	94	3	14	83	60	24	16	57	22	21	0.5
Jordan	5	8	86	6	10	85	3	8	89	3	10	87	29	48	23	29	51	20	0.6
France	9	22	70	2	15	84	0	3	97	0	6	94	14	66	20	14	85	1	0.3
Austria	19	36	46	6	24	70	1	6	93	2	8	90	4	26	70	62	34	4	0.4
Romania	9	58	33	7	51	42	5	29	66	8	24	68	7	45	48	7	55	38	0.3
Germany	20	44	36	3	21	76	0	2	98	2	15	84	0	15	85	32	65	3	0.1
Italy	7	8	86	9	10	81	3	4	93	3	4	93	5	18	77	17	77	6	0.6
Albania	6	7	87	5	13	83	5	4	91	7	8	85	6	62	31	9	81	10	0.5
Greece	1	5	94	1	5	94	0	5	95	0	4	96	11	68	21	22	62	15	0.2
Turkey	1	6	93	1	5	94	0	2	98	0	2	98	6	73	21	7	79	14	0.1
Argentina	m	m	m	13	34	52	1	13	86	1	10	89	10	35	55	18	46	36	m



Countries and economies are ranked in descending order of the average index.
Source: OECD, PISA 2012 Database, Table IV.4.1.

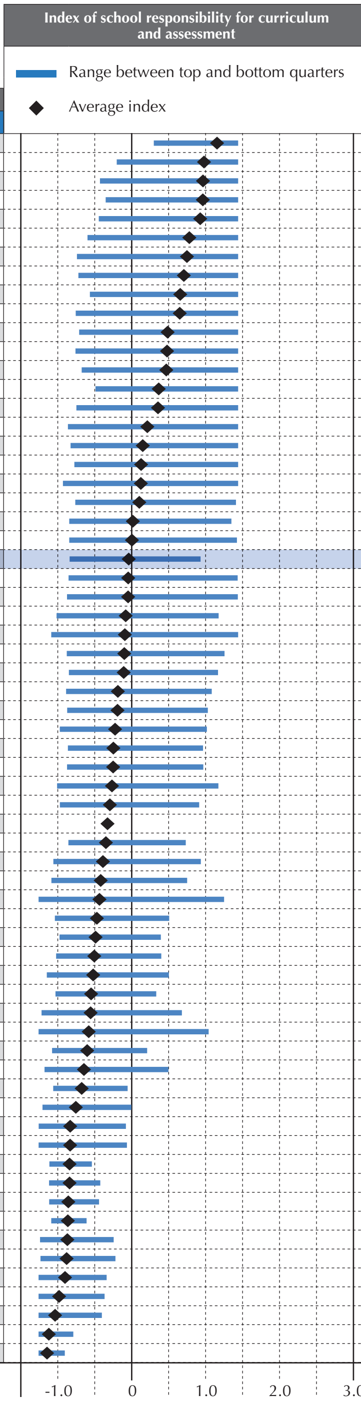
■ Figure 2.8 ■

School autonomy over curricula and assessments

Percentage of students in schools whose principals reported that only "principals and/or teachers", only "regional and/or national education authority", or both "principals and/or teachers" and "regional and/or national education authority", or "school governing board" has/have a considerable responsibility for the following tasks:

- A Establishing student assessment policies
- B Choosing which textbooks are used
- C Determining course content
- D Deciding which courses are offered

- 1 Only "principals and/or teachers"
- 2 Both "principals and/or teachers" and "regional and/or national education authority", or "school governing board"
- 3 Only "regional and/or national education authority"



	A			B			C			D			Variability in the index
	1	2	3	1	2	3	1	2	3	1	2	3	S.D.
Japan	98	2	0	89	7	4	89	7	4	90	6	4	0.7
Thailand	42	54	4	47	53	0	76	24	1	63	37	0	0.7
Netherlands	95	5	0	93	7	0	91	7	2	75	25	0	0.8
Hong Kong-China	87	13	0	84	16	0	76	24	0	63	37	0	0.8
United Kingdom	62	38	0	100	0	0	83	14	3	70	30	0	0.8
Macao-China	66	34	0	81	19	0	66	33	2	36	61	2	0.9
Czech Republic	58	42	0	90	10	0	74	26	0	78	22	0	1.0
Korea	69	29	2	50	50	0	76	20	3	60	36	5	0.9
Lithuania	34	65	1	54	46	0	54	36	10	48	51	1	0.9
Indonesia	82	16	2	80	16	4	73	16	11	62	22	16	1.0
Estonia	39	61	1	70	30	0	35	62	2	48	52	0	0.9
Slovak Republic	76	22	2	68	27	5	61	35	3	56	42	3	1.0
New Zealand	58	38	4	96	4	0	71	28	1	71	29	0	0.9
Poland	57	43	0	82	18	0	83	17	0	36	33	31	0.8
Italy	79	21	0	90	10	0	60	27	12	29	55	15	0.9
Chinese Taipei	27	65	9	67	33	0	54	43	3	30	63	7	0.9
Iceland	63	36	0	77	23	0	51	37	11	39	52	9	1.0
Australia	56	40	3	81	18	0	36	48	16	63	36	2	0.9
Chile	46	51	3	57	36	7	37	35	27	31	60	9	1.0
Ireland	47	51	2	82	18	0	32	40	28	24	75	1	0.8
Hungary	35	64	1	57	43	0	26	59	14	10	65	25	0.9
Israel	57	42	1	45	50	5	47	49	4	30	64	5	0.9
OECD average	47	41	13	65	27	8	40	36	24	36	46	18	0.8
Finland	50	40	10	89	11	0	34	42	24	49	41	10	0.9
Denmark	38	51	11	57	43	0	45	47	8	20	70	10	0.9
Colombia	18	73	9	65	29	6	43	34	23	40	38	22	0.9
Peru	59	32	9	47	16	36	53	28	19	35	26	39	1.0
France	35	50	15	75	25	0	45	38	17	45	37	18	0.9
Belgium	56	40	4	83	15	2	30	45	25	25	62	13	0.8
Latvia	44	52	5	61	38	1	22	40	38	33	54	14	0.8
Germany	40	58	2	40	58	2	21	51	28	52	45	3	0.8
Russian Federation	17	73	10	45	52	3	19	64	17	30	61	10	0.8
Sweden	43	53	3	94	6	0	33	48	20	24	42	33	0.8
Singapore	17	83	0	26	72	2	20	66	15	20	75	5	0.8
Albania	46	22	32	81	15	4	35	30	34	33	35	32	0.9
Austria	38	38	23	60	40	0	35	39	26	10	72	18	0.8
Liechtenstein	54	46	0	16	78	6	6	74	20	9	78	13	0.9
Slovenia	41	55	4	55	44	1	25	63	12	20	73	8	0.7
United States	16	68	16	25	60	15	15	58	26	30	62	8	0.9
Brazil	20	58	22	73	25	2	30	35	35	13	33	54	0.8
United Arab Emirates	35	33	32	28	26	45	26	24	50	25	27	48	1.0
Spain	39	37	24	80	19	1	26	31	43	23	34	42	0.7
Canada	25	58	17	44	43	13	19	39	42	46	51	3	0.7
Argentina	59	33	8	81	18	1	24	36	41	6	24	69	0.7
Romania	22	55	24	38	42	20	26	43	31	16	57	26	0.7
Norway	35	35	29	85	14	1	23	43	34	15	35	51	0.7
Shanghai-China	25	69	6	21	40	39	20	48	32	18	53	29	0.8
Tunisia	27	28	45	29	9	63	28	15	58	29	10	62	1.0
Switzerland	47	42	11	37	38	25	17	46	37	11	61	28	0.6
Costa Rica	41	20	39	69	14	17	20	11	69	13	12	75	0.8
Portugal	18	63	19	79	21	0	6	28	66	10	72	18	0.5
Kazakhstan	34	49	17	16	33	52	7	39	55	16	65	19	0.6
Uruguay	13	45	43	25	41	34	8	33	59	5	42	53	0.6
Montenegro	65	2	33	20	2	78	24	2	75	23	9	68	0.6
Luxembourg	6	49	44	17	69	14	5	65	30	11	71	18	0.4
Bulgaria	18	56	26	45	55	1	8	32	60	2	58	40	0.4
Croatia	17	45	38	49	45	7	10	45	45	4	23	73	0.4
Serbia	51	42	7	34	54	12	5	34	61	0	15	85	0.2
Mexico	33	32	35	51	18	31	12	12	75	4	11	85	0.5
Malaysia	25	23	52	18	9	73	7	9	84	45	28	28	0.6
Qatar	5	48	47	17	46	36	10	44	45	14	40	45	0.5
Viet Nam	12	23	65	20	18	61	8	15	77	18	17	65	0.5
Jordan	14	27	58	6	6	87	7	4	89	6	13	81	0.6
Turkey	2	11	87	4	38	58	4	7	89	4	44	52	0.3
Greece	29	10	61	5	6	89	2	3	95	4	3	93	0.3

Countries and economies are ranked in descending order of the average index.
Source: OECD, PISA 2012 Database, Table IV.4.3.

School choice

Students in some school systems are assigned to attend their neighborhood school. However, in recent decades, reforms in many countries have tended to give greater choice to parents and students, to enable them to choose the schools that meet their children's educational needs or preferences. On the premise that students and parents have adequate information and choose schools based on academic criteria or program quality, the competition for schools creates incentives for institutions to organize programs and teaching in ways that better meet diverse student requirements and interests, thus reducing the cost of failure and mismatches. In some school systems this competition has financial stakes for schools, with schools not only competing for enrolment, but also for funding. This could be through direct public funding of independently managed institutions, based on student enrolments or student credit-hours. Another model is to give money to students and their families (through scholarships or vouchers, for example) to spend on the public or private educational institutions of their choice.

The degree of competition among schools is one way to measure school choice. Competition among schools is intended to provide incentives for schools to innovate and create more effective learning environments.

According to principal's responses, 76% of students attend schools competing with at least one other school for enrolment across OECD countries. The same figure is also reported for the United States. Among OECD countries, only in Iceland, Finland, Norway and Switzerland do less than 50% of students attend schools that compete with other schools for enrolment. In contrast, in Australia, Belgium, Japan, Korea, the Netherlands, New Zealand, and the United Kingdom, over 90% of students attend schools that compete with other schools for enrolment (OECD 2013d, Table IV4.4).

Cross-country correlations in PISA do not show a relationship between the degree of competition and student performance (OECD, 2013d: Table IV.1.4). In 28 countries and economies, schools that compete for student enrolment with other schools tend to show better performance than schools that do not compete, before accounting for schools' socio-economic intake, though this is not the case in the United States. However once the socio-economic status and demographic background of the schools and students are taken into account, only in the Czech Republic, Estonia, Macao-China and Montenegro do schools that compete for students tend to perform better on average (OECD 2013d,Table IV.1.12c).

On the other hand, the results indicate a weak negative relationship between the degree of competition and equity. Among OECD countries, systems with more competition among schools tend to show a stronger impact of students' socio-economic status on their performance in mathematics. Caution is advised when interpreting this result, as the observed relationship could be affected by a few outliers.[9] But this finding is consistent with research showing that school choice – and, by extension, school competition – is related to greater levels of segregation in the school system, which may have adverse consequences for equity in learning opportunities and outcomes.

Public and private schools

Schooling mainly takes places in public institutions. These are defined by PISA as schools managed directly or indirectly by a public education authority, government agency, or governing board appointed by government or elected by public franchise. With an increasing variety of education opportunities, programs and providers, governments are forging new partnerships to mobilize resources for education and to design new policies that allow the different stakeholders to participate more fully and to share costs and benefits more equitably. Private education is not only a way of mobilizing resources from a wider range of funding sources; it is sometimes also regarded as a way of making education more cost-effective. Publicly financed schools are not necessarily also publicly managed. Instead, governments can transfer funds to public and private educational institutions according to various allocation mechanisms.

Across OECD countries, 18% of students are enrolled in privately managed schools that are either privately or government funded, although in many countries government authorities retain significant control over these schools, including the power to shut down non-performing schools. Enrolment in privately managed schools exceeds 50% of 15-year-old students in Chile, Ireland and the Netherlands, and between 35% and 50% in Australia, Korea and the United Kingdom. In contrast, in Iceland, Israel, Norway and Turkey, more than 98% of students attend schools that are publicly managed (OECD 2013d, Table IV.4.7).

Across OECD countries and all countries and economies that participated in PISA 2012, the percentage of students enrolled in private schools is not related to a system's overall performance (OECD 2013d, Table IV.1.4). At the school level, privately managed schools show a performance advantage of 28 points on the PISA reading scale, on average across OECD countries, although in the United States the advantage is smaller and not statistically significant. However, once the socio-economic background of students and schools is accounted for, public schools come out with a slight advantage of seven points, on average across OECD countries. In the United States, once socio-economic background is accounted for, public schools show superior performance compared with private schools, amounting to 27 points.

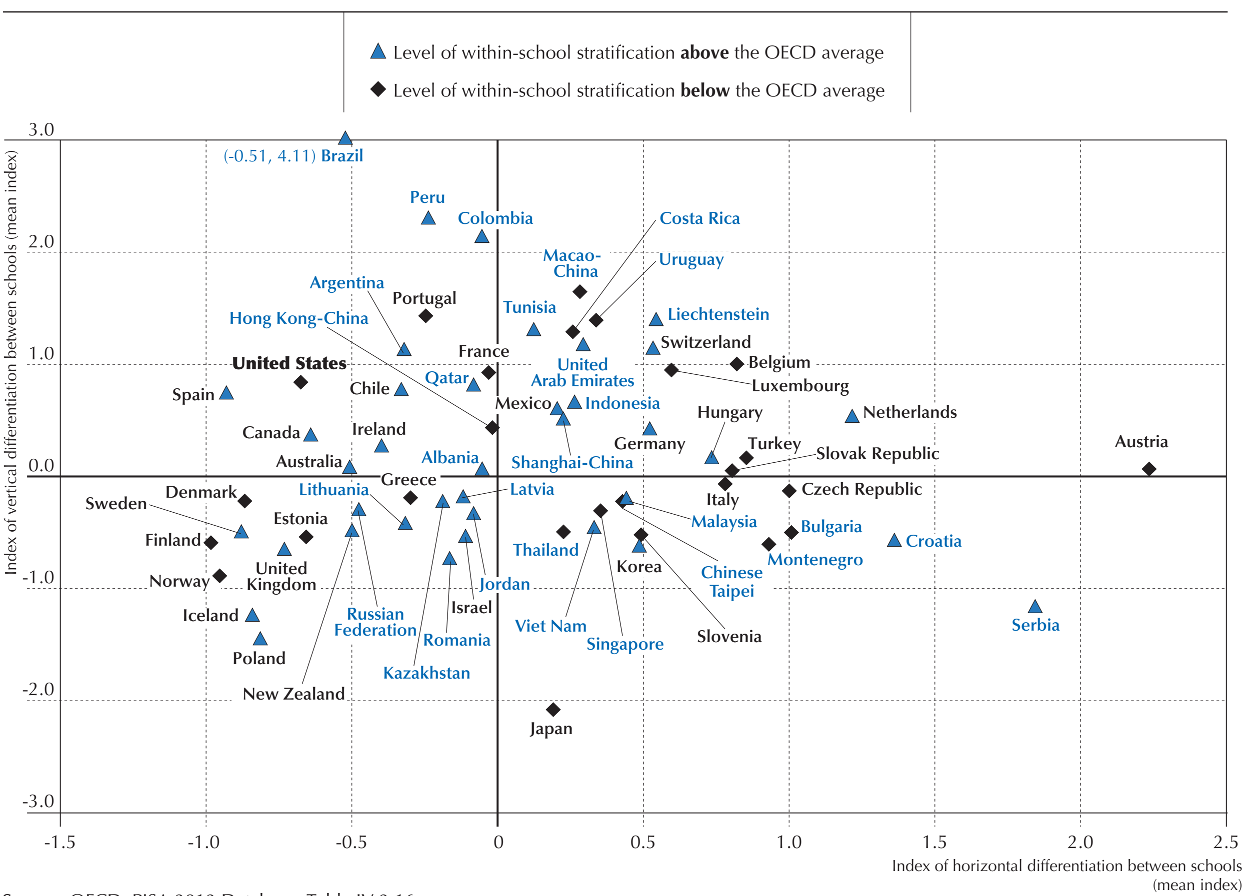
Selection of students into schools, grades and programs

While teaching and learning are at the heart of schooling, they are supported by a complex organization responsible for everything from selecting and admitting students to schools and classrooms to evaluating their progress; formulating curricula; promoting successful approaches to teaching and learning; creating incentives to motivate students and teachers; and deciding on the distribution of financial, material and human resources – all with the aim of providing quality education. This section looks at how school systems are organized to allocate students to programs, schools and classes. It considers horizontal and vertical forms of stratification.

Horizontal stratification can be adopted by the school system or by individual schools, groups students according to their interests and/or performance. School systems make decisions on offering specific programs (vocational or academic, for example), setting the age at which students are admitted into these programs, and determining the extent to which students' academic records are used to select students for their schools. Individual schools make decisions about whether to transfer students out of the school because of poor performance, behavioral problems or special needs, and whether to group students in classes according to ability. Vertical stratification refers to the ways in which students progress through school as they become older. Even though the student population is differentiated into grade levels in practically all schools that participate in PISA, in some countries, all 15-year-old students attend the same grade level, while in other countries they are dispersed throughout various grade levels as a result of policies governing the age of entrance into the school system and/or grade repetition

■ Figure 2.9 ■

Vertical and horizontal stratification



Source: OECD, PISA 2012 Database, Table IV.2.16.

Figure 2.9 plots countries according to the degree to which there is horizontal and vertical stratification. Countries and economies in the top right quadrant are those that have higher levels of vertical and horizontal (between-school) stratification than the OECD average. Countries and economies in the bottom left quadrant of the chart are those that have lower levels of vertical and horizontal (between-school) stratification than the OECD average. This shows the United States to be above average vertical stratification but below average horizontal stratification.

PISA shows that the degree of school systems' vertical stratification tends to be negatively related to how equitable the education outcomes are. In systems where 15-year-old students are found in different grade levels, the impact of students' socio-economic status on their academic performance is stronger than in systems with less vertical stratification. Across OECD countries, 32% of the variation in the impact of students' socio-economic status on their mathematics performance can be explained by differences in the degree of vertical stratification within the system, after accounting for per capita GDP (OECD 2013d, Table IV.1.1).[10] On the other hand, across OECD countries there is no clear relationship between vertical stratification and average performance of the system after accounting for per capita GDP.[11]

Grade repetition is a feature of vertical stratification in some countries. Requiring that students repeat grades implies some cost, not only the direct cost of providing an additional year of education, but also the opportunity cost to society of delaying that student's entry into the labor market by at least one year (OECD, 2011). Across OECD countries, an average of 12% of students reported that they had repeated at least on grade in primary, lower secondary or upper secondary school. The rate of grade repetition in the United States is similar to the OECD average. Grade repetition tends to be negatively related to equity, and this is especially obvious when the relationship is examined across OECD countries (OECD 2013d, Figure IV.1.4). Across OECD countries, 20% of the variation in the impact of students' socio-economic status on their mathematics performance can be explained by differences in the proportion of students who repeated a grade, even after accounting for per capita GDP. At the same time, across OECD countries, grade repetition is unrelated to the system's overall performance.

In comprehensive school systems, all 15-year-old students follow the same program, while in differentiated school systems, students are streamed horizontally into different programs. Differentiated systems must also decide at which age students will be sorted into different programs. On average across OECD countries, school systems begin selecting students for different programs at the age of 14. However, this varies greatly across countries, from age 10 in Austria and Germany, to age 16 in a number of OECD countries, including the United States. When it comes to the number of school types or distinct educational programs available to 15-year-old students, the United States follows the most common practice among OECD countries of offering only one type of school or program. This is in contrast to the Czech Republic, the Netherlands and the Slovak Republic where five or more programs are available.

In general, this type of between-school horizontal stratification is unrelated to a system's average performance. The exception is that, across all participating countries and economies, systems that group students within schools for all classes based on their ability tend to have lower performance after accounting for per capita GDP (with a partial correlation coefficient of -0.25). Horizontal stratification is also negatively related to equity in education opportunities. The impact of the socio-economic status of students and/or schools on performance is stronger in school systems that sort students into different tracks, where students are grouped into different tracks at an early age, where more students attend vocational programs, where more students attend academically selective schools, or where more students attend schools that transfer low-performing students or students with behavior problems to another school. Across OECD countries, 47% of the variation in the impact of socio-economic status of students and schools on students' mathematics performance can be explained by differences in the ages at which students are selected into different programs, even after accounting for per capita GDP (OECD 2013d, Table IV.1.1).

The age at which stratification begins may be closely associated with the impact of socio-economic status on performance because the frequency and the nature of student selections/transitions differ between early- and late-stratified systems. In systems that stratify students early, students might be selected more than once before the age of 15. When students are older, more information on individual students is available, and decisions on selecting and sorting students into certain tracks are thus better informed and teachers and parents have enough information to make more objective judgments. In addition, students are more dependent upon their parents and their parents' resources when they are younger. In systems that stratify students early, parents with more advantaged socio-economic status may be in a better position to promote their children's chances than disadvantaged parents.

As expected, schools that select students based on students' academic performance tend to show better average performance, even after accounting for the socio-economic status and demographic background of students and schools and various other

school characteristics, on average across OECD countries (OECD 2013d, Table IV.1.12c). However, the performance of a school system overall is not better if it has a greater proportion of academically selective schools. In fact, in systems with more academically selective schools, the impact of the socio-economic status of students and schools on student performance is stronger (OECD 2013d, Table IV.1.1).

ASSESSMENT AND ACCOUNTABILITY ARRANGEMENTS

Education standards

The shift in public and government concern towards a focus on outcomes has, in many countries, led to the establishment of quality standards for educational institutions. In most OECD countries, evaluation and assessment systems concentrate not only on the students, but also on teachers and school leaders. Countries pursue a range of approaches to standard setting, from defining broad education goals to formulating precise performance expectations in well-defined subject areas. The use of performance data to improve teaching and learning has also expanded in recent years (OECD, 2013g).

Education standards have influenced OECD education systems in various ways. They have helped to establish rigorous, focused and coherent content at all grade levels. They have also reduced overlaps in curricula across grades and reduced the variation in the way curricula are implemented across classrooms. Standards facilitate the co-ordination of policy drivers ranging from curricula to teacher training and reduce inequity in curricula across socio-economic groups. The United States has suffered from wide discrepancies between state standards and test scores that have led to non-comparable results. These discrepancies often mean that a school's fate depends more than anything else on where it is located and, perhaps even more importantly, that students across the United States are not equally well prepared to compete in the United States labor market. The establishment of the "Common Core Standards" in the United States could be a step towards addressing these problems and indeed 46 states and the District of Columbia have signed up to the standards. Chapter 4 of this report will turn to examine the extent to which PISA mathematics corresponds to the Common Core State Standards for Mathematics.

PISA 2012 collected data on the nature of accountability systems and the ways in which the resulting information was used and made available to various stakeholders and the general public.

Examinations

Countries and economies implement different policies to evaluate their students' performance. System-wide evaluations can generally be classified into those that do not have direct consequences for students (assessments) and those that do (examinations). Assessments can be used to take stock of students' performance in order to make decisions on future instruction or to summarize performance for information purposes. Although assessments can be used to, for example, decide on allocation of resources to low-performing schools or tailor instruction to low-performing students, assessment results do not have direct tangible consequences for students. Results from examinations, by contrast, can be used to determine students' progression to higher levels of education (for example, the transition from lower to upper secondary school), selection into different curricular programs (for example, into vocational or academic programs), or selection into university programs. Assessments and examinations provide students with benchmarks, and, in the case of examinations, with incentives to work hard in school in order to pass.

All PISA-participating countries and economies have an assessment or examination system in place.[12] However, the characteristics of these systems can vary greatly in terms of the emphasis given to examinations versus assessments and the stages of a students' schooling when they are implemented.

Countries and economies can be grouped into four categories of assessment-and-examination systems as shown in Figure 2.10. A first group of countries and economies tends to have assessments at the lower secondary level and national examinations at the upper secondary level, with few tertiary fields of study requiring a special examination for admission. A second group of countries and economies tends to have national examinations at both the upper and secondary levels. A third group of countries and economies tends to rely on not only national examinations, but also other types of examinations or on other types of examinations only. The fourth group of countries and economies tends to have no examinations at the lower or upper secondary level, but a large number of tertiary fields of study require examinations.[13] The United States is one of only three OECD countries in the third group that has a mix of national or other non-national examinations in lower or upper secondary.

Among OECD countries, 12 school systems conduct national examinations in lower secondary school and 21 do so in upper secondary school; all partner countries and economies conduct them in upper secondary school. At the lower

secondary level, these examinations are, in all cases, used to certify students' graduation or grade completion. In Norway and Poland, these examinations are used to determine access to selective upper secondary schools; and in Scotland, Norway and Ireland they are used to select students into certain programs, courses or tracks in upper secondary school. In all OECD countries, the results from these examinations are shared directly with students. They are also shared with an external audience in addition to education authorities, with school administrators (except in Italy), and directly with parents (except in Germany). In all OECD countries except in general programs in Poland, upper secondary examinations are used to certify completion or graduation. Except in the United States and in pre-vocational and vocational programs in Hungary and Spain, they are also used to determine students' access to tertiary education. In 15 OECD countries upper secondary examinations are also used to determine student selection for fields of study at the tertiary level (OECD 2013d, Tables IV.4.22 and IV.4.23).

▪ Figure 2.10 ▪

Country profile: Assessments and examinations

Assessment in lower secondary, national exams in upper secondary, few fields requiring tertiary exams	Only national exams in lower and upper secondary	National or other non-national examinations in lower or upper secondary	No national or other examinations, most fields requiring tertiary exams
Australia	Albania	Belgium (Fr. Comm.)	Austria
Croatia	Bulgaria	Liechtenstein	Belgium (Fl. Comm.)
Czech Republic	Denmark	Montenegro	Brazil
England (UK)	Estonia	Norway	Chile
Finland	France	Qatar	Colombia
Hong Kong-China	Germany	United Arab Emirates	Greece
Hungary	Indonesia	**United States**	Iceland
Israel	Ireland		Japan
Luxembourg	Italy		Korea
Scotland (UK)	Jordan		Macao-China
Singapore	Latvia		Mexico
Slovak Republic	Lithuania		Peru
Tunisia	Malaysia		Spain
	Netherlands		Sweden
	Poland		Turkey
	Portugal		Uruguay
	Romania		
	Russian Federation		
	Shanghai-China		
	Chinese Taipei		
	Thailand		
	Viet Nam		

Source: OECD, PISA 2012 Database, Tables IV.4.20, IV.4.21, IV.4.22, IV.4.23, IV.4.24, IV.4.25 and IV.4.26.

The use of achievement data beyond school

While performance data in the United States are often used for purely accountability purposes, other countries tend to give greater weight to using them to guide intervention, reveal best practices and identify shared problems. Where school performance is systematically assessed, the primary purpose is often not to support an evaluation of public services or to support market mechanisms in the allocation of resources; rather it is to reveal best practices and identify common problems in order to encourage teachers and schools to develop more supportive and productive learning environments. To achieve this, many education systems try to develop assessment and accountability systems that include progressive learning targets that explicitly describe the steps that learners follow as they become more proficient, and define what a student should know and be able to do at each level of advancement. The trend among OECD countries is towards multi-layered, coherent assessment systems from classrooms to schools to regional to national to international levels. These assessment systems support improvement of learning at all levels of the system and are increasingly performance-based. They add value for teaching and learning by providing information that can be acted on by students, teachers, and administrators and are part of a comprehensive and well-aligned learning system that includes syllabi, associated instructional materials, matching exams, professional scoring and teacher training.

Achievement data can offer accountability when they are shared with stakeholders beyond the school, teachers, partners and students. School principals were asked to report on whether achievement data are posted publicly, or tracked over time by an administrative authority. On average across OECD countries, 45% of students are in schools whose principals reported that achievement data are posted publicly. In the United States as well as in the Netherlands, New Zealand,

Sweden and the United Kingdom over 80% of students attend such schools, while in Argentina, Austria, Belgium, Japan, Macao-China, Shanghai-China and Uruguay, fewer than 10% of students do (OECD 2013d, Table IV.4.31).

In most countries, tracking achievement data over time seems to be a more common practice than posting such data publicly. In the United States, virtually all students are in schools whose principals reported that achievement data are tracked over time by an administrative authority (the OECD average is 72%). In contrast only 7% of students in Japan attend such schools (OECD 2013d, Figure IV.4.13 and Table IV.4.31).

Quality assurance

Schools also use measures other than student assessments to monitor the quality of the education they provide. PISA 2012 asked school principals to report on whether their schools use various measures related to quality assurance and improvement. Analysis of these responses shows that systems that seek written feedback from students regarding lessons, teachers or resources tend to perform better. Across OECD countries, some 10% of the variation in the impact of students' socio-economic status on their mathematics performance can be accounted for by differences in the degree to which systems use this approach, after accounting for per capita GDP (OECD 2013d, Table IV.1.4). Among OECD countries, in the Netherlands, New Zealand and Turkey, over 85% of students attend schools whose principals reported that the school seeks written feedback from students. In the United States, some 59% of students attend such schools, a figure that is close to the OECD average (OECD 2013d, Figure IV.4.14 and Table IV.4.32).

On the other quality assurance measures that school principals were asked about, these all seem to be in relatively frequent use in the schools attended by 15-year-olds in the United States.

Across all countries and economies that participated in PISA 2012, but not across OECD countries on average, the extent to which schools provide opportunities for teacher mentoring is related to equity. In the systems where more schools provide teacher mentoring, students' socio-economic status has less impact on their performance, both before and after accounting for per capita GDP (OECD 2013d, Table IV.1.4).

RESOURCES

Effective school systems require the right combination of trained and talented personnel, adequate educational resources and facilities, and motivated students ready to learn. But performance on international comparisons cannot simply be tied to money, since only Luxembourg spends more per student than the United States. The results for the United States reflect a range of inefficiencies. That point is reinforced by the fact that the United States does relatively well by international standards in both the Trends in International Mathematics and Science Study (TIMSS) and the Progress in International Reading Literacy Study (PIRLS), which compare children in primary school. Given the country's wealth, that would be expected; the problem is that as they get older, children in the United States make less progress each year than children in the best-performing countries do. It is noteworthy that spending patterns in many of the world's successful education systems are markedly different from those in the United States. Successful systems such as Canada, Finland and Shanghai-China invest money where the challenges are greatest, rather than making the resources that are devoted to schools dependent on the wealth of the local communities in which schools are located; and they put in place incentives and support systems that attract the most talented school teachers into the most difficult classrooms. They have often reformed the traditional and bureaucratic systems they inherited for recruiting and training teachers and leaders, paying and rewarding them and shaping their incentives, both short term and long term.

Research usually shows a weak relationship between educational resources and student performance, with more variation explained by the quality of human resources (i.e. teachers and school principals) than by material and financial resources, particularly among industrialized nations. The generally weak relationship between resources and performance observed in past research is also seen in PISA.

Financial resources

A first glance at PISA results gives the impression that high-income countries and economies – and those that are able to and spend more on education – have better student performance. But, as noted in Box 2.1, higher expenditure on education is only predictive of higher PISA mathematics scores among countries and economies whose cumulative expenditure per student is below USD 50 000. This is not the case among high-income countries and economies, which include most OECD countries.

Thus, among these higher-income countries and economies, it is common to find some with substantially different levels of spending per student yet similar mathematics performance. For example, the United States and the Slovak Republic score 481 points in mathematics, but the United States' cumulative expenditure per student is more than double that of the Slovak Republic. Similarly, countries with similar levels of expenditure can perform very differently. For example, Italy and Singapore both have a cumulative expenditure per student of roughly USD 85 000, but while Italy scored 485 points in mathematics in PISA 2012, Singapore scored 573 points (OECD 2013d, Figure IV.1.8).

Moreover, analysis of PISA data shows that there is no relationship between increases in expenditure and changes in performance between 2003 and 2012.

A notable finding from PISA is that high-performing systems tend to prioritize higher salaries for teachers, especially in high-income countries (see OECD 2013d, Figure IV.1.10). Among countries whose per capita GDP is more than USD 20 000, including most OECD countries, systems that pay teachers more relative to national income tend to perform better in mathematics. The correlation between these two factors among 33 high-income countries is 0.30, and the correlation is 0.40 among 32 high-income countries excluding Qatar.[14] In contrast, among countries whose per capita GDP is under USD 20 000, a system's overall academic performance is unrelated to its teachers' salaries, suggesting that a host of other resources (material infrastructure, instructional materials, transportation, etc.) also need to be improved until they reach a certain threshold, after which improvements in material resources no longer benefit student performance, but improvements in human resources (through higher teachers' salaries, for example) do.[15]

Human resources

As with spending per student, the quantity of human resources tends to be unrelated to the academic performance or equity of school systems, after accounting for the level of national income.[16] Of course, a school system that lacks quality teachers, infrastructure and textbooks will almost certainly perform at lower levels than other systems. In fact, at the school level, teacher shortage appears to be related to poorer performance in most countries. In 33 countries and economies, including the United States, schools where a higher share of principals reported that teacher shortages hinder learning tend to show lower performance (OECD 2013d, Table IV.3.10). However, the degree of teacher shortage is related to the amount of other resources allocated to schools and to schools' socio-economic intake. The United States is one of a group of countries where advantaged and disadvantaged schools show particularly wide differences in the level of teacher shortages.

Material resources

The educational resources available in a school tend to be related to the system's overall performance as well as schools' average level of performance. Furthermore, it is shown that high-performing systems tend to allocate resources more equitably between socio-economically advantaged and disadvantaged schools.

After accounting for per capita GDP, 33% of the variation in mathematics performance across OECD countries can be explained by differences in principals' responses to questions about the adequacy of science laboratory equipment, instructional materials (e.g. textbooks), computers for instruction, Internet connectivity, computer software for instruction, and library materials (OECD 2013d, Table IV.1.2). In the schools attended by 15-year-olds in the United States, principals are relatively positive about the adequacy of educational resources in their schools, though they were least positive about the adequacy of computers used for instruction.

How resources are allocated to disadvantaged and advantaged schools is also related to systems' levels of performance. In higher-performing systems, principals in socio-economically advantaged and disadvantaged schools reported similar levels of quality of physical infrastructure and schools' educational resources, both across OECD countries and across all countries and economies that participated in PISA 2012 (OECD 2013d, Table IV.1.3). In the United States, the gap between the perceptions of principals in disadvantaged versus advantaged schools is large, with those from advantaged schools being far more positive. It is also important that, within school systems, much of the relationship between school resources and student performance is closely associated with schools' socio-economic and demographic profiles. This suggests the need for more consideration on how to distribute resources for schools more equitably.

Time resources

The average amount of time spent learning in regular mathematics lessons is positively related to student performance at the school level. Even after accounting for the socio-economic status and demographic profile of students and schools and various other school characteristics, in 15 countries and economies schools with more mathematics learning time

tended to perform better in mathematics, though this is not the case in the United States (OECD 2013d, Table IV.1.12c). However, at the system level, across all OECD countries and all countries and economies that participated in PISA 2012, there is no clear pattern linking a system's overall mathematics performance and whether students in that system spend more time in regular mathematics classes or not (OECD 2013d, Table IV.1.2).[17] Since learning outcomes are the product of both the quantity and the quality of instruction time, this suggests that differences in the quality of instruction time between countries blur the relationship between the quantity of instruction time and student performance.

Schools whose students spend more hours on homework or other study set by teachers tend, on average, to perform better, even after accounting for the socio-economic status and demographic background of students and schools and various other school characteristics, and this is true in the United States (OECD 2013d, Tables IV.1.8b, IV.1.8c and IV.1.12b). This is not an obvious finding, since one might expect that lower-performing students spend more time doing homework. However, there may be other factors, such as higher-performing schools requiring more homework from their students. At the system level, the average number of hours that students spend on homework or other study tends to be unrelated to overall performance (OECD 2013d, Table IV.1.2).

Whether and for how long students are enrolled in pre-primary education is another important aspect of time resources invested in education. Many of the inequalities that exist within school systems are already present when students first enter formal schooling and persist as students progress.

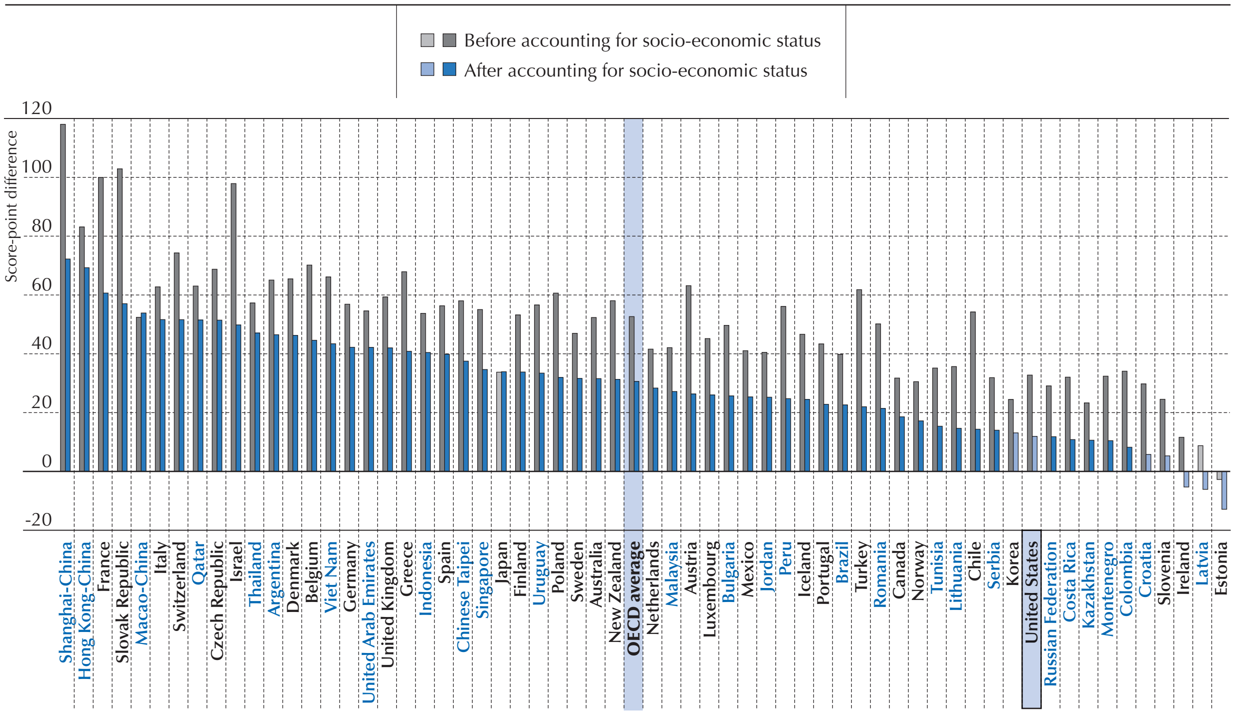
Students who had attended pre-primary education tend to perform better at the age of 15 than those who had not attended pre-primary education (Figure 2.11). In almost all countries, though not in the United States, this performance advantage remains even after accounting for socio-economic background. In general, across OECD countries, 74% of students reported that they had attended pre-primary education for more than one year – close to the figure for the United States – while in 24 participating countries and economies, over 80% of students reported that they had attended pre-primary education for more than one year (OECD 2013d, Table IV.3.33). The relationship between attendance at pre-primary education and performance is also apparent at the school level. In 19 countries and economies, schools with more students who had attended pre-primary education for more than one year tend to show better average performance, though this was not the case for the United States, (OECD 2013d, Table IV.1.12c). At the system level, across all countries and economies participating in PISA, there is also a relationship between the proportion of students who had attended pre-primary education for more than one year and overall performance in mathematics. Some 32% of the variation in mathematics performance across all countries and economies can be explained by the difference in the percentage of students who attended pre-primary education for more than one year, after accounting for per capita GDP (OECD 2013d, Table IV.1.2). However, across OECD countries, there is no clear relationship.

While these results underline the importance of pre-primary education, international comparisons of children in primary school show that the United States does well by international standards. However, as noted earlier, as they get older, these children make less progress each year than children in many other countries. In other words, more pre-primary education can only be part of the solution; quality of provision must also be ensured.

■ Figure 2.11 ■

Difference in mathematics performance, by attendance at pre-primary school

Between students who attended pre-primary school for more than one year and those who had not attended



Note: Score-point differences that are statistically significant are marked in a darker tone.
Countries and economies are ranked in descending order of the score-point difference in mathematics performance between students who reported that they had attended pre-primary school (ISCED 0) for more than one year and those who had not attended pre-primary school, after accounting for socio-economic status.
Source: OECD, PISA 2012 Database, Table II.4.12.

When the impact of pre-primary education attendance on reading performance at age 15 is compared between different socio-economic backgrounds, no significant difference is found between students from socio-economically disadvantaged and advantaged backgrounds (OECD 2013b, Table II.4.13). Students benefit equally from pre-primary education attendance whether they are socio-economically advantaged or disadvantaged in 29 OECD countries and 24 partner countries and economies.

The next chapter examines the performance of 15-year-old students in a finer level of detail by looking at the different items in the PISA test. This reveals relative strengths and weaknesses of students, with important messages for teaching in the United States.

■ Figure 2.12 ■

Comparing countries' and economies' performance in mathematics

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
613	Shanghai-China	
573	Singapore	
561	Hong Kong-China	Chinese Taipei, Korea
560	Chinese Taipei	Hong Kong-China, Korea
554	Korea	Hong Kong-China, Chinese Taipei
538	Macao-China	Japan, Liechtenstein
536	Japan	Macao-China, Liechtenstein, Switzerland
535	Liechtenstein	Macao-China, Japan, Switzerland
531	Switzerland	Japan, Liechtenstein, Netherlands
523	Netherlands	Switzerland, Estonia, Finland, Canada, Poland, Viet Nam
521	Estonia	Netherlands, Finland, Canada, Poland, Viet Nam
519	Finland	Netherlands, Estonia, Canada, Poland, Belgium, Germany, Viet Nam
518	Canada	Netherlands, Estonia, Finland, Poland, Belgium, Germany, Viet Nam
518	Poland	Netherlands, Estonia, Finland, Canada, Belgium, Germany, Viet Nam
515	Belgium	Finland, Canada, Poland, Germany, Viet Nam
514	Germany	Finland, Canada, Poland, Belgium, Viet Nam
511	Viet Nam	Netherlands, Estonia, Finland, Canada, Poland, Belgium, Germany, Austria, Australia, Ireland
506	Austria	Viet Nam, Australia, Ireland, Slovenia, Denmark, New Zealand, Czech Republic
504	Australia	Viet Nam, Austria, Ireland, Slovenia, Denmark, New Zealand, Czech Republic
501	Ireland	Viet Nam, Austria, Australia, Slovenia, Denmark, New Zealand, Czech Republic, France, United Kingdom
501	Slovenia	Austria, Australia, Ireland, Denmark, New Zealand, Czech Republic
500	Denmark	Austria, Australia, Ireland, Slovenia, New Zealand, Czech Republic, France, United Kingdom
500	New Zealand	Austria, Australia, Ireland, Slovenia, Denmark, Czech Republic, France, United Kingdom
499	Czech Republic	Austria, Australia, Ireland, Slovenia, Denmark, New Zealand, France, United Kingdom, Iceland
495	France	Ireland, Denmark, New Zealand, Czech Republic, United Kingdom, Iceland, Latvia, Luxembourg, Norway, Portugal
494	United Kingdom	Ireland, Denmark, New Zealand, Czech Republic, France, Iceland, Latvia, Luxembourg, Norway, Portugal
493	Iceland	Czech Republic, France, United Kingdom, Latvia, Luxembourg, Norway, Portugal
491	Latvia	France, United Kingdom, Iceland, Luxembourg, Norway, Portugal, Italy, Spain
490	Luxembourg	France, United Kingdom, Iceland, Latvia, Norway, Portugal
489	Norway	France, United Kingdom, Iceland, Latvia, Luxembourg, Portugal, Italy, Spain, Russian Federation, Slovak Republic, United States
487	Portugal	France, United Kingdom, Iceland, Latvia, Luxembourg, Norway, Italy, Spain, Russian Federation, Slovak Republic, United States, Lithuania
485	Italy	Latvia, Norway, Portugal, Spain, Russian Federation, Slovak Republic, United States, Lithuania
484	Spain	Latvia, Norway, Portugal, Italy, Russian Federation, Slovak Republic, United States, Lithuania, Hungary
482	Russian Federation	Norway, Portugal, Italy, Spain, Slovak Republic, United States, Lithuania, Sweden, Hungary
482	Slovak Republic	Norway, Portugal, Italy, Spain, Russian Federation, United States, Lithuania, Sweden, Hungary
481	**United States**	Norway, Portugal, Italy, Spain, Russian Federation, Slovak Republic, Lithuania, Sweden, Hungary
479	Lithuania	Portugal, Italy, Spain, Russian Federation, Slovak Republic, United States, Sweden, Hungary, Croatia
478	Sweden	Russian Federation, Slovak Republic, United States, Lithuania, Hungary, Croatia
477	Hungary	Spain, Russian Federation, Slovak Republic, United States, Lithuania, Sweden, Croatia, Israel
471	Croatia	Lithuania, Sweden, Hungary, Israel
466	Israel	Hungary, Croatia
453	Greece	Serbia, Turkey, Romania
449	Serbia	Greece, Turkey, Romania, Bulgaria
448	Turkey	Greece, Serbia, Romania, Cyprus[1, 2], Bulgaria
445	Romania	Greece, Serbia, Turkey, Cyprus[1, 2], Bulgaria
440	Cyprus[1, 2]	Turkey, Romania, Bulgaria
439	Bulgaria	Serbia, Turkey, Romania, Cyprus[1, 2], United Arab Emirates, Kazakhstan
434	United Arab Emirates	Bulgaria, Kazakhstan, Thailand
432	Kazakhstan	Bulgaria, United Arab Emirates, Thailand
427	Thailand	United Arab Emirates, Kazakhstan, Chile, Malaysia
423	Chile	Thailand, Malaysia
421	Malaysia	Thailand, Chile
413	Mexico	Uruguay, Costa Rica
410	Montenegro	Uruguay, Costa Rica
409	Uruguay	Mexico, Montenegro, Costa Rica
407	Costa Rica	Mexico, Montenegro, Uruguay
394	Albania	Brazil, Argentina, Tunisia
391	Brazil	Albania, Argentina, Tunisia, Jordan
388	Argentina	Albania, Brazil, Tunisia, Jordan
388	Tunisia	Albania, Brazil, Argentina, Jordan
386	Jordan	Brazil, Argentina, Tunisia
376	Colombia	Qatar, Indonesia, Peru
376	Qatar	Colombia, Indonesia
375	Indonesia	Colombia, Qatar, Peru
368	Peru	Colombia, Indonesia

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.
Source: OECD, PISA 2012 Database.

■ Figure 2.13 ■

Comparing countries' and economies' performance in reading

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
570	Shanghai-China	
545	Hong Kong-China	Singapore, Japan, Korea
542	Singapore	Hong Kong-China, Japan, Korea
538	Japan	Hong Kong-China, Singapore, Korea
536	Korea	Hong Kong-China, Singapore, Japan
524	Finland	Ireland, Chinese Taipei, Canada, Poland, Liechtenstein
523	Ireland	Finland, Chinese Taipei, Canada, Poland, Liechtenstein
523	Chinese Taipei	Finland, Ireland, Canada, Poland, Estonia, Liechtenstein
523	Canada	Finland, Ireland, Chinese Taipei, Poland, Liechtenstein
518	Poland	Finland, Ireland, Chinese Taipei, Canada, Estonia, Liechtenstein, New Zealand, Australia, Netherlands, Viet Nam
516	Estonia	Chinese Taipei, Poland, Liechtenstein, New Zealand, Australia, Netherlands, Viet Nam
516	Liechtenstein	Finland, Ireland, Chinese Taipei, Canada, Poland, Estonia, New Zealand, Australia, Netherlands, Belgium, Switzerland, Macao-China, Viet Nam, Germany
512	New Zealand	Poland, Estonia, Liechtenstein, Australia, Netherlands, Belgium, Switzerland, Macao-China, Viet Nam, Germany, France
512	Australia	Poland, Estonia, Liechtenstein, New Zealand, Netherlands, Belgium, Switzerland, Macao-China, Viet Nam, Germany, France
511	Netherlands	Poland, Estonia, Liechtenstein, New Zealand, Australia, Belgium, Switzerland, Macao-China, Viet Nam, Germany, France, Norway
509	Belgium	Liechtenstein, New Zealand, Australia, Netherlands, Switzerland, Macao-China, Viet Nam, Germany, France, Norway
509	Switzerland	Liechtenstein, New Zealand, Australia, Netherlands, Belgium, Macao-China, Viet Nam, Germany, France, Norway
509	Macao-China	Liechtenstein, New Zealand, Australia, Netherlands, Belgium, Switzerland, Viet Nam, Germany, France, Norway
508	Viet Nam	Poland, Estonia, Liechtenstein, New Zealand, Australia, Netherlands, Belgium, Switzerland, Macao-China, Germany, France, Norway, United Kingdom, United States
508	Germany	Liechtenstein, New Zealand, Australia, Netherlands, Belgium, Switzerland, Macao-China, Viet Nam, France, Norway, United Kingdom
505	France	New Zealand, Australia, Netherlands, Belgium, Switzerland, Macao-China, Viet Nam, Germany, Norway, United Kingdom, United States
504	Norway	Netherlands, Belgium, Switzerland, Macao-China, Viet Nam, Germany, France, United Kingdom, United States, Denmark
499	United Kingdom	Viet Nam, Germany, France, Norway, United States, Denmark, Czech Republic
498	**United States**	Viet Nam, France, Norway, United Kingdom, Denmark, Czech Republic, Italy, Austria, Hungary, Portugal, Israel
496	Denmark	Norway, United Kingdom, United States, Czech Republic, Italy, Austria, Hungary, Portugal, Israel
493	Czech Republic	United Kingdom, United States, Denmark, Italy, Austria, Latvia, Hungary, Spain, Luxembourg, Portugal, Israel, Croatia
490	Italy	United States, Denmark, Czech Republic, Austria, Latvia, Hungary, Spain, Luxembourg, Portugal, Israel, Croatia, Sweden
490	Austria	United States, Denmark, Czech Republic, Italy, Latvia, Hungary, Spain, Luxembourg, Portugal, Israel, Croatia, Sweden
489	Latvia	Czech Republic, Italy, Austria, Hungary, Spain, Luxembourg, Portugal, Israel, Croatia, Sweden
488	Hungary	United States, Denmark, Czech Republic, Italy, Austria, Latvia, Spain, Luxembourg, Portugal, Israel, Croatia, Sweden, Iceland
488	Spain	Czech Republic, Italy, Austria, Latvia, Hungary, Luxembourg, Portugal, Israel, Croatia, Sweden
488	Luxembourg	Czech Republic, Italy, Austria, Latvia, Hungary, Spain, Portugal, Israel, Croatia, Sweden
488	Portugal	United States, Denmark, Czech Republic, Italy, Austria, Latvia, Hungary, Spain, Luxembourg, Israel, Croatia, Sweden, Iceland, Slovenia
486	Israel	United States, Denmark, Czech Republic, Italy, Austria, Latvia, Hungary, Spain, Luxembourg, Portugal, Croatia, Sweden, Iceland, Slovenia, Lithuania, Greece, Turkey, Russian Federation
485	Croatia	Czech Republic, Italy, Austria, Latvia, Hungary, Spain, Luxembourg, Portugal, Israel, Sweden, Iceland, Slovenia, Lithuania, Greece, Turkey
483	Sweden	Italy, Austria, Latvia, Hungary, Spain, Luxembourg, Portugal, Israel, Croatia, Iceland, Slovenia, Lithuania, Greece, Turkey, Russian Federation
483	Iceland	Hungary, Portugal, Israel, Croatia, Sweden, Slovenia, Lithuania, Greece, Turkey
481	Slovenia	Portugal, Israel, Croatia, Sweden, Iceland, Lithuania, Greece, Turkey, Russian Federation
477	Lithuania	Israel, Croatia, Sweden, Iceland, Slovenia, Greece, Turkey, Russian Federation
477	Greece	Israel, Croatia, Sweden, Iceland, Slovenia, Lithuania, Turkey, Russian Federation
475	Turkey	Israel, Croatia, Sweden, Iceland, Slovenia, Lithuania, Greece, Russian Federation
475	Russian Federation	Israel, Sweden, Slovenia, Lithuania, Greece, Turkey
463	Slovak Republic	
449	Cyprus[1,2]	Serbia
446	Serbia	Cyprus[1,2], United Arab Emirates, Chile, Thailand, Costa Rica, Romania, Bulgaria
442	United Arab Emirates	Serbia, Chile, Thailand, Costa Rica, Romania, Bulgaria
441	Chile	Serbia, United Arab Emirates, Thailand, Costa Rica, Romania, Bulgaria
441	Thailand	Serbia, United Arab Emirates, Chile, Costa Rica, Romania, Bulgaria
441	Costa Rica	Serbia, United Arab Emirates, Chile, Thailand, Romania, Bulgaria
438	Romania	Serbia, United Arab Emirates, Chile, Thailand, Costa Rica, Bulgaria
436	Bulgaria	Serbia, United Arab Emirates, Chile, Thailand, Costa Rica, Romania
424	Mexico	Montenegro
422	Montenegro	Mexico
411	Uruguay	Brazil, Tunisia, Colombia
410	Brazil	Uruguay, Tunisia, Colombia
404	Tunisia	Uruguay, Brazil, Colombia, Jordan, Malaysia, Indonesia, Argentina, Albania
403	Colombia	Uruguay, Brazil, Tunisia, Jordan, Malaysia, Indonesia, Argentina
399	Jordan	Tunisia, Colombia, Malaysia, Indonesia, Argentina, Albania, Kazakhstan
398	Malaysia	Tunisia, Colombia, Jordan, Indonesia, Argentina, Albania, Kazakhstan
396	Indonesia	Tunisia, Colombia, Jordan, Malaysia, Argentina, Albania, Kazakhstan
396	Argentina	Tunisia, Colombia, Jordan, Malaysia, Indonesia, Albania, Kazakhstan
394	Albania	Tunisia, Jordan, Malaysia, Indonesia, Argentina, Kazakhstan, Qatar, Peru
393	Kazakhstan	Jordan, Malaysia, Indonesia, Argentina, Albania, Qatar, Peru
388	Qatar	Albania, Kazakhstan, Peru
384	Peru	Albania, Kazakhstan, Qatar

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.
Source: OECD, PISA 2012 Database.

■ Figure 2.14 ■
Comparing countries' and economies' performance in science

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Mean score	Comparison country/ economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
580	Shanghai-China	
555	Hong Kong-China	Singapore, Japan
551	Singapore	Hong Kong-China, Japan
547	Japan	Hong Kong-China, Singapore, Finland, Estonia, Korea
545	Finland	Japan, Estonia, Korea
541	Estonia	Japan, Finland, Korea
538	Korea	Japan, Finland, Estonia, Viet Nam
528	Viet Nam	Korea, Poland, Canada, Liechtenstein, Germany, Chinese Taipei, Netherlands, Ireland, Australia, Macao-China
526	Poland	Viet Nam, Canada, Liechtenstein, Germany, Chinese Taipei, Netherlands, Ireland, Australia, Macao-China
525	Canada	Viet Nam, Poland, Liechtenstein, Germany, Chinese Taipei, Netherlands, Ireland, Australia
525	Liechtenstein	Viet Nam, Poland, Canada, Germany, Chinese Taipei, Netherlands, Ireland, Australia, Macao-China
524	Germany	Viet Nam, Poland, Canada, Liechtenstein, Chinese Taipei, Netherlands, Ireland, Australia, Macao-China
523	Chinese Taipei	Viet Nam, Poland, Canada, Liechtenstein, Germany, Netherlands, Ireland, Australia, Macao-China
522	Netherlands	Viet Nam, Poland, Canada, Liechtenstein, Germany, Chinese Taipei, Ireland, Australia, Macao-China, New Zealand, Switzerland, United Kingdom
522	Ireland	Viet Nam, Poland, Canada, Liechtenstein, Germany, Chinese Taipei, Netherlands, Australia, Macao-China, New Zealand, Switzerland, United Kingdom
521	Australia	Viet Nam, Poland, Canada, Liechtenstein, Germany, Chinese Taipei, Netherlands, Ireland, Macao-China, Switzerland, United Kingdom
521	Macao-China	Viet Nam, Poland, Liechtenstein, Germany, Chinese Taipei, Netherlands, Ireland, Australia, Switzerland, United Kingdom
516	New Zealand	Netherlands, Ireland, Switzerland, Slovenia, United Kingdom
515	Switzerland	Netherlands, Ireland, Australia, Macao-China, New Zealand, Slovenia, United Kingdom, Czech Republic
514	Slovenia	New Zealand, Switzerland, United Kingdom, Czech Republic
514	United Kingdom	Netherlands, Ireland, Australia, Macao-China, New Zealand, Switzerland, Slovenia, Czech Republic, Austria
508	Czech Republic	Switzerland, Slovenia, United Kingdom, Austria, Belgium, Latvia
506	Austria	United Kingdom, Czech Republic, Belgium, Latvia, France, Denmark, United States
505	Belgium	Czech Republic, Austria, Latvia, France, United States
502	Latvia	Czech Republic, Austria, Belgium, France, Denmark, United States, Spain, Lithuania, Norway, Hungary
499	France	Austria, Belgium, Latvia, Denmark, United States, Spain, Lithuania, Norway, Hungary, Italy, Croatia
498	Denmark	Austria, Latvia, France, United States, Spain, Lithuania, Norway, Hungary, Italy, Croatia
497	**United States**	Austria, Belgium, Latvia, France, Denmark, Spain, Lithuania, Norway, Hungary, Italy, Croatia, Luxembourg, Portugal
496	Spain	Latvia, France, Denmark, United States, Lithuania, Norway, Hungary, Italy, Croatia, Portugal
496	Lithuania	Latvia, France, Denmark, United States, Spain, Norway, Hungary, Italy, Croatia, Luxembourg, Portugal
495	Norway	Latvia, France, Denmark, United States, Spain, Lithuania, Hungary, Italy, Croatia, Luxembourg, Portugal, Russian Federation
494	Hungary	Latvia, France, Denmark, United States, Spain, Lithuania, Norway, Italy, Croatia, Luxembourg, Portugal, Russian Federation
494	Italy	France, Denmark, United States, Spain, Lithuania, Norway, Hungary, Croatia, Luxembourg, Portugal
491	Croatia	France, Denmark, United States, Spain, Lithuania, Norway, Hungary, Italy, Luxembourg, Portugal, Russian Federation, Sweden
491	Luxembourg	United States, Lithuania, Norway, Hungary, Italy, Croatia, Portugal, Russian Federation
489	Portugal	United States, Spain, Lithuania, Norway, Hungary, Italy, Croatia, Luxembourg, Russian Federation, Sweden
486	Russian Federation	Norway, Hungary, Croatia, Luxembourg, Portugal, Sweden
485	Sweden	Croatia, Portugal, Russian Federation Iceland
478	Iceland	Sweden, Slovak Republic, Israel
471	Slovak Republic	Iceland, Israel, Greece, Turkey
470	Israel	Iceland, Slovak Republic, Greece, Turkey
467	Greece	Slovak Republic, Israel, Turkey
463	Turkey	Slovak Republic, Israel, Greece
448	United Arab Emirates	Bulgaria, Chile, Serbia, Thailand
446	Bulgaria	United Arab Emirates, Chile, Serbia, Thailand, Romania, Cyprus[1, 2]
445	Chile	United Arab Emirates, Bulgaria, Serbia, Thailand, Romania
445	Serbia	United Arab Emirates, Bulgaria, Chile, Thailand, Romania
444	Thailand	United Arab Emirates, Bulgaria, Chile, Serbia, Romania
439	Romania	Bulgaria, Chile, Serbia, Thailand, Cyprus[1, 2]
438	Cyprus[1, 2]	Bulgaria, Romania
429	Costa Rica	Kazakhstan
425	Kazakhstan	Costa Rica, Malaysia
420	Malaysia	Kazakhstan, Uruguay, Mexico
416	Uruguay	Malaysia, Mexico, Montenegro, Jordan
415	Mexico	Malaysia, Uruguay, Jordan
410	Montenegro	Uruguay, Jordan, Argentina
409	Jordan	Uruguay, Mexico, Montenegro, Argentina, Brazil
406	Argentina	Montenegro, Jordan, Brazil, Colombia, Tunisia, Albania
405	Brazil	Jordan, Argentina, Colombia, Tunisia
399	Colombia	Argentina, Brazil, Tunisia, Albania
398	Tunisia	Argentina, Brazil, Colombia, Albania
397	Albania	Argentina, Colombia, Tunisia
384	Qatar	Indonesia
382	Indonesia	Qatar, Peru
373	Peru	Indonesia

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.
Source: OECD, PISA 2012 Database.

Notes

1. Though rank 26 is the best estimate, due to sampling and measurement error the rank could be between 23 and 29.

2. Though rank 17 is the best estimate, due to sampling and measurement error the rank could be between 14 and 20.

3. Though rank 21 is the best estimate, due to sampling and measurement error the rank could be between 17 and 25.

4. A socio-economically disadvantaged school is one whose students' mean socio-economic status is statistically significantly below the mean socio-economic status of the country. See Table II.2.11 of OECD, 2013b.

5. All figures shown in purchasing power parities.

6. This is measured by the *PISA index of economic, social and cultural status* of students. The index has an average of 0 and a standard deviation of 1 for OECD countries. The index value for the most disadvantaged quarter of students is -1.14 for the United States and -1.15 for the OECD average. The index value for the entire student population is 0.17 for the United States and 0.00 for the OECD average.

7. No such data are available for the United States.

8. Resilient students are those who come from a socio-economically disadvantaged background and perform much higher than would be predicted by their background. To identify these students, first the relationship between performance and socio-economic background across all students participating in the PISA 2012 assessment is established. Then the actual performance of each disadvantaged student is compared with the performance predicted by the average relationship among students from similar socio-economic backgrounds across countries. This difference is defined as the student's residual performance. A disadvantaged student is classified as resilient if his or her residual performance is found to be among the top quarter of students' residual performance from all countries.

9. Across OECD countries, the correlation between the degree of competition and equity is 0.33 (significant at the 10% level), while it is 0.23 after excluding Norway, where there is less school competition than in other countries (i.e. the degree of school competition is 35% in Norway, while it varies from 42% to 94% in other OECD countries).

10. The percentage is obtained by squaring the partial correlation coefficient and then multiplying it by 100.

11. Partial correlation coefficients are -0.36 among all participating countries and economies (significant at the 5% level).

12. Information is available for all OECD countries except Canada, New Zealand and Slovenia. Information is available for all participating partner countries and economies except Argentina, Costa Rica, Kazakhstan and Serbia. Switzerland and Turkey do not have information on the existence of assessments so they are excluded from the analysis.

13. These groups are created using a cluster analysis with the Ward method, which groups countries and economies to minimize the variance within each cluster, using data available in Table IV.3.4 in OECD, 2013d. Variables that entered the analyses are: the existence of national assessments in lower secondary and upper secondary schools, the percentage of students taking national examinations in lower and upper secondary general programs, the percentage of students taking other examinations in lower and upper secondary general programs, and the percentage of tertiary fields of study requiring a non-secondary school examination for access. For those countries and economies where the percentage of students taking the examinations is unavailable, if examinations are compulsory, a percentage of 100 is used (Viet Nam), and if not compulsory, a percentage of 50 is used (Australia, upper secondary education). When the percentage of students taking other examinations is missing, a percentage value of 0 is used if no information on other examinations is provided (Australia, Slovenia, Korea, Turkey, Romania, Tunisia and Viet Nam); if these examinations do exist, then a value of 50 is used (Japan). When the number of fields of study requiring a tertiary examination is missing, a value of 0 is used (Tunisia).

14. Among OECD countries, the correlation is 0.32.

15. The correlation is -0.22 among 17 countries and economies whose per capita GDP is less than USD 20 000.

16. Statistically significant coefficients in Table IV.1.2 in OECD, 2013d are mainly the result of outliers. For example, the correlation between the student-teacher ratio and performance is -0.48 across OECD countries, but it is 0.09 after excluding two countries with extreme student-teacher ratios (31 in Mexico and 22 in Chile, while the average ranges from 8 to 18 in other OECD countries).

17. Across OECD countries, the correlation between mathematics performance and average learning time in regular mathematics lessons is -0.30 (significant at the 10% level), but this is mainly because of outliers.

References

OECD (2013), *PISA 2012 Database.*
http://pisa2012.acer.edu.au/

OECD (2013a), *PISA 2012 Results: What Students Know and Can Do (Volume I): Student Performance in Mathematics, Reading and Science*, Vol. I, OECD Publishing.

OECD (2013b), *PISA 2012 Results: Excellence through Equity (Volume II): Giving Every Student the Chance to Succeed*, OECD Publishing.
http://dx.doi.org/10.1787/9789264201132-en

OECD (2013c), *PISA 2012 Results: Ready to Learn (Volume III): Students' Engagement, Drive and Self-Beliefs*, OECD Publishing.
http://dx.doi.org/10.1787/9789264201170-en

OECD (2013d), *PISA 2012 Results: What Makes a School Successful (Volume IV): Resources, Policies and Practices*, OECD Publishing.
http://dx.doi.org/10.1787/9789264201156-en

OECD (2013e), Education at a Glance: OECD Indicators, OECD Publishing.
http://dx.doi.org/10.1787/eag-2013-en

OECD (2013f), OECD Skills Outlook 2013: First Results from the Survey of Adult Skills, *OECD* Publishing.
http://dx.doi/org/10.1787/9789264204256-en

OECD (2013g), *Synergies for Better Learning: An International Perspective on Evaluation and Assessment*, OECD Publishing.
http://dx.doi/org/10.1787/9789264190658-en

OECD (2012), *Learning beyond Fifteen: Ten Years after PISA*, PISA, OECD Publishing.
http://dx.doi/org/10.1787/9789264172104-en

OECD (2011), "When Students Repeat Grades or Are Transferred Out of School: What Does it Mean for Education Systems?",
PISA in Focus, No. 6, PISA, OECD Publishing.
http://dx.doi/org/10.1787/9789264201118-en

OECD (2010a), *Pathways to Success: How Knowledge and Skills at Age 15 Shape Future Lives in Canada*, OECD Publishing.
http://dx.doi/org/10.1787/9789264081925-en

OECD (2010b), *The High Cost of Low Educational Performance - The Long-Run Economic Impact of Improving PISA Outcomes,*
OECD Publishing.
http://dx.doi/org/10.1787/9789264077485-en

3

Strengths and Weaknesses of American Students in Mathematics

This chapter looks in more detail at American students' strengths and weaknesses in the PISA 2012 mathematics assessment. It examines the success rates of students on individual items of the test, compared with the success of students in OECD countries on average and in comparison with five comparator or reference countries/economies. The analysis points to aspects of mathematics teaching that might be strengthened in order to improve the performance of the United States.

INTRODUCTION

Having presented the performance of 15-year-old students in the United States in the previous chapter, this chapter looks in more detail at their strengths and weaknesses in the PISA 2012 mathematics assessment. It examines the success rates of students on individual items of the test, compared with the success (measured by solution rates) of students in OECD countries on average and in comparison with five comparator or reference countries/economies. The five countries chosen for comparison with the United States are: two top performing Asian countries, Shanghai-China and Korea; two European countries performing significantly above the OECD average, the Netherlands and Germany; and one of the United States' neighboring countries, Canada.

This analysis examines the performance of the United States students in terms of the percentage of students who correctly answered each of the 84 mathematics items that were administered in the United States as part of PISA 2012. It compares the performance of students in the United States with both the OECD average and with the performance of the five reference countries/economies to identify specific relative strengths and relative weaknesses of the country's 15-year-olds. The analysis identifies the so-called "conspicuous items", that is those items where the United States' students performed unexpectedly well or unexpectedly badly compared with their overall distance from the OECD average or from the reference countries. Altogether the analysis reveals 33 such conspicuous items, in 16 of which the United States was notably strong and 17 in which it was notably weak. The analysis shows that the relatively strong items are mostly easy ones whereas the relatively weak items are much more demanding. This analysis identifies certain patterns, with clusters of conspicuous items that have similar cognitive requirements. Some of the items from these patterns are then analyzed more deeply in order to understand the clusters better. An analysis of student solutions helps to clarify further the strengths and weaknesses of the students.

What is PISA mathematics about? The conceptual core of PISA mathematics is *mathematical literacy*. According to the PISA 2012 mathematics framework (see OECD 2013, p. 25) this concept means "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens." In short, mathematical literacy describes one's capacity to use mathematics in a well-founded manner in order to solve real world problems, where "real world" means "the physical, social and mental world" (Freudenthal, 1983), including the mathematical world itself. Annex A1 gives an overview of the framework for assessing mathematics in PISA 2012.

OVERALL SOLUTION RATES

Across all of the 84 mathematics items administered in PISA 2012 in the United States, on average the United States has a significantly lower solution rate than the OECD average. The solution rate of the United States is 43.78% against an average of all OECD countries of 47.47%. In only 18 items did the United States reach a higher solution rate than the OECD average (see Table A1.1 for a full list of the item solution rates). Other countries with which the United States has to compete economically reached much higher averages in PISA 2012 mathematics. Table 3.1 shows the average percentage success rates of these reference countries.

■ Table 3.1 ■

Average solution rates in PISA 2012 mathematics (%)

United States	OECD average	Canada	Germany	Netherlands	Korea	Shanghai-China
43.78	47.47	52.16	51.34	53.50	58.76	69.44

These lower overall solution rates of course reflect the relative overall performance in the PISA 2012 mathematics assessment of 15-year-olds in the United States as reported in Chapter 2. The remainder of this chapter will examine the solution rates item by item to reveal where the United States does relatively better or relatively worse.

SELECTION OF CONSPICUOUS ITEMS

The analysis seeks to identify so-called "conspicuous items", which stand out as indicating particular strengths and weaknesses among United States' students, compared with those of the average of all OECD countries and the five

reference systems: Canada, Germany, Korea, the Netherlands and Shanghai-China. Different approaches can be followed to identify such conspicuous items. The simplest approach would be simply to compare solution rates. However, such an approach would be distorted when comparing items with relatively low or high solution rates overall. For instance a difference of 10% between solution rates is less notable when the two solution rates are 95% and 85% than when they are 55% and 45% – in other words, the relationship between getting items correct and the items' difficulty is not linear (see Table A1.1 for comparative solution rates for all 84 mathematics items).

Therefore it is necessary to transform the percentage solution rates into a linear metric. This can be achieved by transforming the percentages into "logits". This transformation has the effect of "stretching out" very low and very high solution rates in comparison with solution rates close to 50%. A logit value of 0 means that the item has a solution rate of 50%, positive logits mean higher solution rates and negative logits mean lower solution rates.

How U.S. students compare with the OECD average

As a first step the logit differences between the United States and the average of all OECD countries were compared in order to identify the items with notably different solution rates. Significant items are those with a difference of at least one standard deviation (both sides)[1] between the United States and the OECD average. The analysis also refers to the categorization of the items within the PISA 2012 mathematics framework, an overview of which is given in Annex A1 of this report. This categorization, for instance, identifies the content area of mathematics to which the item belongs (Quantity, Uncertainty and data, Change and relationships, Space and shape) and also the mathematical process that is called on in order to answer the question (Formulating situations mathematically, Employing mathematical concepts, facts, procedures and reasoning and Interpreting, applying and evaluating mathematical outcomes).

Relative strengths

Table 3.2 shows the 12 conspicuously strong items identified by this method, all with significant positive differences in the logits of the United States compared with the average of all OECD countries. It is remarkable that there are no items from the content area "Space and shape" and only one item from "Quantity". The others are either "Change and relationships" or "Uncertainty and data" items. It is equally remarkable that only one item belongs to the process category "Formulate", whereas seven items stem from the category "Interpret" and six items from "Employ".

■ Table 3.2 ■

Relative strengths of the United States compared with the average of all OECD countries (in logits)

Name	Item Code	Logit USA	Logit average for all OECD countries	Difference	Content	Process
Charts	PM918Q01	2.43	1.92	0.51	Uncertainty and data	Interpret
Bike rental	PM998Q02	1.43	0.92	0.51	Change and relationships	Interpret
Speeding fines	PM909Q01	2.57	2.13	0.45	Quantity	Interpret
Transport	PM420Q01T	0.32	0.00	0.31	Uncertainty and data	Interpret
Thermometer cricket	PM446Q01	1.08	0.78	0.30	Change and relationships	Formulate
Medicine doses	PM954Q01	0.86	0.64	0.23	Change and relationships	Employ
Drip rate	PM903Q03	-0.85	-1.06	0.21	Change and relationships	Employ
Carbon tax	PM915Q01	-0.26	-0.40	0.14	Uncertainty and data	Employ
Diving	PM411Q02	-0.04	-0.17	0.13	Uncertainty and data	Interpret
Employment data	PM982Q02	-0.70	-0.81	0.11	Uncertainty and data	Employ
Carbon dioxide	PM828Q02	0.35	0.24	0.11	Uncertainty and data	Employ
Population pyramids	PM155Q01	0.82	0.74	0.08	Change and relationships	Interpret

Relative weaknesses

A further 12 conspicuous items show significant negative differences, indicating the relative weaknesses of students in the United States (Table 3.3). In contrast to the relatively strong items, seven of the items where United States students are weak are in the domain of "Space and shape". There is only one item (PM955Q02 i.e. item 2 of the unit "Migration") which calls on students' skills in interpreting a mathematical problem.

■ Table 3.3 ■

Relative weaknesses of the United States compared with the average of all OECD countries (in logits)

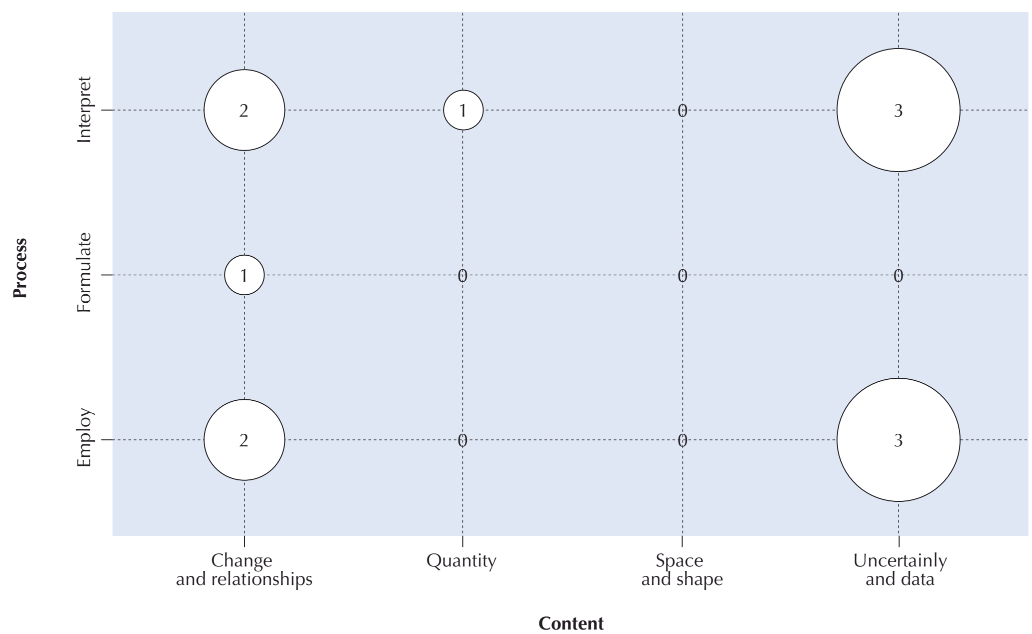
Name	Item Code	Logit USA	Logit average for all OECD countries	Difference	Content	Process
Wheelchair basketball	PM00KQ02	-2.59	-1.75	-0.84	Space and shape	Formulate
Arches	PM943Q02	-3.67	-2.89	-0.79	Space and shape	Formulate
Migration	PM955Q03	-2.76	-1.99	-0.76	Uncertainty and data	Employ
Migration	PM955Q02	-1.41	-0.65	-0.76	Uncertainty and data	Interpret
Running tracks	PM406Q02	-2.33	-1.59	-0.74	Space and shape	Formulate
Computer game	PM800Q01	1.35	2.03	-0.68	Quantity	Employ
Carbon tax	PM915Q02	0.12	0.77	-0.65	Change and relationships	Employ
The fence	PM464Q01T	-1.81	-1.17	-0.64	Space and shape	Formulate
Map	PM305Q01	-0.14	0.42	-0.56	Space and shape	Employ
Running tracks	PM406Q01	-1.61	-1.07	-0.54	Space and shape	Employ
An advertising column	PM00GQ01	-2.87	-2.34	-0.53	Space and shape	Formulate
Sauce	PM924Q02	0.05	0.55	-0.50	Quantity	Formulate

Conspicuous items by "Content" and "Process"

In order to get a first impression of the similarities and differences of these conspicuous items, it is helpful to have a look at the "Content" and "Process" categories of these items. Figures 3.1 and 3.2 examine the correlations between the contents and the processes of the conspicuous items.

■ Figure 3.1 ■

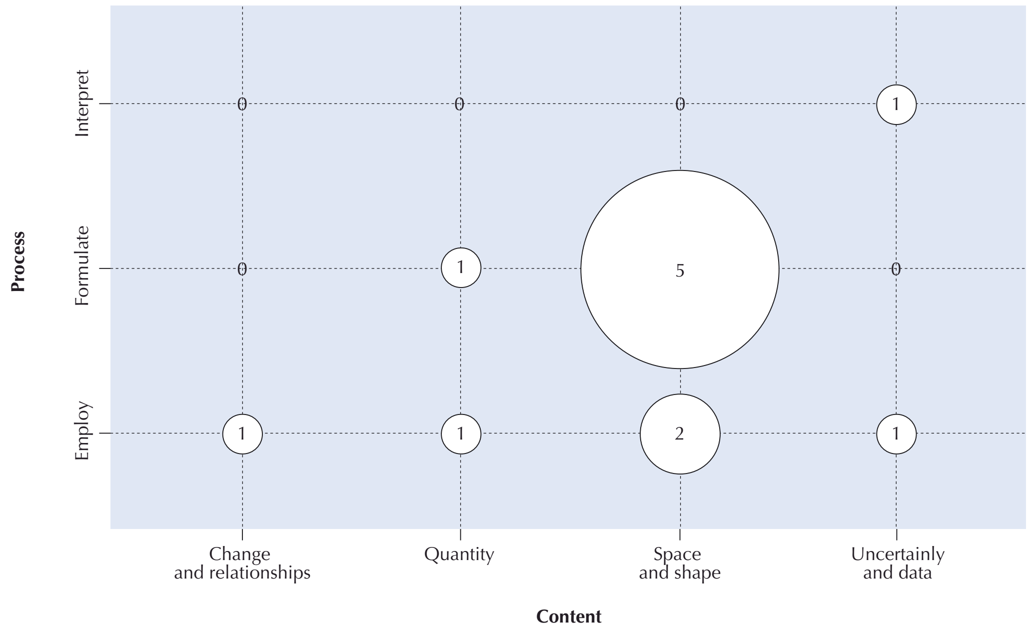
Bubble chart of *process* and *content* for the relative strengths



It is apparent that the overlap between these bubble charts for the strengths and weaknesses is almost negligible. Obviously, these items fall into several different clusters. In other words, the strengths of the United States students lie primarily in items which require employing or interpreting mathematical concepts or facts and are either from the mathematical content area "Change and relationships" or from "Uncertainty and data". In contrast, their weaknesses lie primarily in items which require formulating or employing mathematical concepts or facts from the mathematical content area "Space and shape".

■ Figure 3.2 ■

Bubble chart of *process* and *content* for the relative weaknesses



Comparison with the reference countries

Table 3.4 compares the United States with the 5 reference countries/economies for the 12 conspicuous items that showed relative strength when compared with the OECD average (from Table 3.2).

Relative strengths

Table 3.4 shows that in none of these items did the United States perform better than Shanghai-China. Additionally there were three items – "Diving 02" (i.e. item 2 of the unit "Diving"), "Carbon dioxide 02" and "Population pyramids 01" – where the United States did not perform better than any of the reference countries. Only in two items ("Charts 01" and "Transport 01") did the United States score considerably better than three out of the five reference countries. Among the comparator countries, the United States performed best in comparison with Germany, performing better than Germany in nine out of the 12 items. It performed better than the Netherlands in six items, better than Korea in three and better than Canada in only two.

■ Table 3.4 ■

Conspicuous items compared with the reference countries (in logits)

Name	Item Code	Logit USA	Logit average for all OECD countries	Logit Canada	Logit Germany	Logit Netherlands	Logit Korea	Logit Shanghai-China
Charts	PM918Q01	2.43	1.92	1.89	1.99	2.25	2.29	2.52
Bike rental	PM998Q02	1.43	0.92	1.73	1.16	2.10	1.39	1.96
Speeding fines	PM909Q01	2.57	2.13	2.82	2.50	3.11	2.67	2.83
Transport	PM420Q01T	0.32	0.00	0.58	0.03	0.31	-0.94	0.43
Thermometer cricket	PM446Q01	1.08	0.78	1.26	0.62	1.00	1.34	2.00
Medicine doses	PM954Q01	0.86	0.64	0.94	0.77	0.36	1.46	2.51
Drip rate	PM903Q03	-0.85	-1.06	-0.53	-1.11	-0.82	-0.15	0.88
Carbon tax	PM915Q01	-0.26	-0.40	-0.38	-0.76	-0.02	0.69	0.98
Diving	PM411Q02	-0.04	-0.17	0.08	-0,01	0.49	0.26	0.90
Employment data	PM982Q02	-0.70	-0.81	-0.58	-0.91	-0.84	-0.22	-0.51
Carbon dioxide	PM828Q02	0.35	0.24	0.51	0.40	0.82	0.68	1.16
Population pyramids	PM155Q01	0.82	0.74	1.06	0.83	1.38	1.24	0.98

Note: Shading indicates the United States performs better than the comparator country.

Relative weaknesses

Considering the 12 items where the United States is relatively weak compared with the OECD average, and comparing these with the comparator countries (Table 3.5), it is evident that in all of these items, the United States performs worse than each of the comparator countries. This is most notable in "Wheelchair basketball 02" and "Map 01", the latter being an item in which all of the other comparator countries had solution rates of more than 50% (positive logits), whereas the solution rate for the United States was less than 50% (negative logit).

■ Table 3.5 ■

Conspicuous items compared with the reference countries (in logits)

Name	Item Code	Logit USA	Logit average for all OECD countries	Logit Canada	Logit Germany	Logit Netherlands	Logit Korea	Logit Shanghai-China
Wheelchair basketball	PM00KQ02	-2.59	-1.75	-1.44	-1.31	-1.53	-1.77	-0.17
Arches	PM943Q02	-3.67	-2.89	-3.14	-2.78	-3.39	-1.42	0.04
Migration	PM955Q03	-2.76	-1.99	-1.74	-1.79	-1.25	-1.16	-0.29
Migration	PM955Q02	-1.41	-0.65	-0.62	-0.56	-0.06	0.25	0.31
Running tracks	PM406Q02	-2.33	-1.59	-1.14	-1.42	-1.21	-0.84	-0.20
Computer game	PM800Q01	1.35	2.03	1.93	1.71	2.02	3.30	3.43
Carbon tax	PM915Q02	0.12	0.77	1.16	1.42	1.12	1.05	2.21
The fence	PM464Q01T	-1.81	-1.17	-0.83	-0.73	-1.18	-0.32	0.40
Map	PM305Q01	-0.14	0.42	0.55	0.68	0.71	0.51	0.94
Running tracks	PM406Q01	-1.61	-1.07	-0.60	-0.80	-0.86	-0.40	0.32
An advertising column	PM00GQ01	-2.87	-2.34	-2.28	-2.06	-1.70	-1.72	-1.52
Sauce	PM924Q02	0.05	0.55	0.43	0.69	1.47	1.01	1.74

Note: See Table 3.3.

Comparison with the average of the reference countries

Further conspicuous items can be found by generating an average logit value for each item, referring only to the five reference countries. This average is compared with the logits of the United States and again, "unexpectedly large" and "unexpectedly small" differences are examined. More precisely, all those items where the difference between those two logits (the United States minus the average of the reference countries) is at least by one standard deviation larger or smaller than the average of all differences[2] are considered conspicuous.

Relative strengths

This comparison reveals four new conspicuous items, highlighted in dark blue. The item "Employment data 01" shows, remarkably enough, a positive value compared with the two Asian countries, the other three items show a positive logit value in comparison with the Netherlands. The results for "Employment data 01" might also reveal one of the rare relative weaknesses of the Asian countries.

■ Table 3.6 ■

Relative strengths of the United States compared with the reference countries (in logits)

Name	Item Code	Logit USA	Logit average for comparator countries	Logit Canada	Logit Germany	Logit Netherlands	Logit Korea	Logit Shanghai-China
Charts	PM918Q01	2.43	2.19	1.89	1.99	2.25	2.29	2.52
Transport	PM420Q01T	0.32	0.08	0.58	0.03	0.31	-0.94	0.43
Employment data	PM982Q01	1.92	1.88	1.97	2.10	2.06	1.76	1.49
Employment data	PM982Q02	-0.70	-0.61	-0.58	-0.91	-0.84	-0.22	-0.51
Thermometer cricket	PM446Q01	1.08	1.25	1.26	0.62	1.00	1.34	2.00
Speeding fines	PM909Q01	2.57	2.79	2.82	2.50	3.11	2.67	2.83
Bike rental	PM998Q02	1.43	1.67	1.73	1.16	2.10	1.39	1.96
Migration	PM955Q01	1.02	1.26	1.33	1.33	0.87	1.11	1.68
Population pyramids	PM155Q01	0.82	1.10	1.06	0.83	1.38	1.24	0.98
Population pyramids	PM155Q04T	0.16	0.47	0.56	0.35	0.04	0.59	0.81
Arches	PM943Q01	0.05	0.37	0.15	0.08	-0.20	0.52	1.30

Note: Shading indicates the United States performs better than the comparator country.

DETECTING PATTERNS

The analysis so far has revealed 33 conspicuous items: 16 items in which the United States is relatively strong (12 in Table 3.4 and four in Table 3.6) and 17 items in which the United States is relatively weak (12 in Table 3.5 and five in Table 3.7). The next question is whether, when considering these 33 items, different items have the same or similar cognitive requirements – are there certain patterns that can be detected? Of course, every single PISA item has its own characteristics. A pattern is thus composed of items that share some common features although other features remain item specific. Seven such patterns were detected, three for relative strengths and four for relative weaknesses of the United States. The vast majority (27) of the conspicuous items fit into one of these patterns.

For each pattern, the following analysis selects one "illustrating item", an item that is particularly typical of this pattern. For some of the patterns, a second item is selected that is also fairly typical and which belongs to the set of publicly released PISA items.[3]

In some of the patterns, certain "demarcating items" are identified. These are items which seem, at first glance, to belong to that pattern, but which in practice require different competencies in order to solve them. A demarcating item for a strength pattern might therefore be relatively harder for U.S. students than the pattern suggests, while one for a weakness pattern might be relatively easier.

Three out of the 33 items do not fit the patterns and therefore are not considered further in the analysis. These are "Population pyramids 04", "Diving 02", and "Sauce 02". Another three are used as demarcating items for some of the patterns: "Arches 01" for a weakness pattern, and "Computer game 01" and "Carbon tax 02" for strength patterns.

■ Table 3.7 ■

Relative weaknesses of the United States compared with the reference countries (in logits)

Name	Item Code	Logit USA	Logit average for comparator countries	Logit Canada	Logit Germany	Logit Netherlands	Logit Korea	Logit Shanghai-China
Arches	PM943Q02	-3.67	-2.14	-3.14	-2.78	-3.39	-1.42	0.04
Migration	PM955Q03	-2.76	-1.24	-1.74	-1.79	-1.25	-1.16	-0.29
Running tracks	PM406Q02	-2.33	-0.96	-1.14	-1.42	-1.21	-0.84	-0.20
Wheelchair basketball	PM00KQ02	-2.59	-1.24	-1.44	-1.31	-1.53	-1.77	-0.17
The fence	PM464Q01T	-1.81	-0.53	-0.83	-0.73	-1,18	-0.32	0.40
Migration	PM955Q02	-1.41	-0.14	-0.62	-0.56	-0.06	0.25	0.31
Carbon tax	PM915Q02	0.12	1.39	1.16	1.42	1.12	1.05	2.21
Roof truss design	PM949Q01T	0.31	1.47	1.15	1.00	1.25	1.76	2.17
Running tracks	PM406Q01	-1.61	-0.47	-0.60	-0.80	-0.86	-0.40	0.32
Spacers	PM992Q03	-2.86	-1.73	-2.35	-2.23	-2.61	-0.98	-0.49
Computer game	PM800Q01	1.35	2.48	1.93	1.71	2.02	3.30	3.43
Thermometer cricket	PM446Q02	-2.87	-1.74	-2.33	-2.55	-1.84	-1.64	-0.36
Revolving door	PM995Q01	-0.12	0.95	0.30	0.58	0.37	1.33	2.16
Tennis balls	PM905Q02	-0.46	0.60	0.30	0.26	0.48	0.54	1.43

This method reveals five new conspicuous items, again highlighted in dark blue.

Relative strengths of the United States

It is remarkable that the following "strength patterns" include the majority of easy PISA items; there are only a few more challenging ones here. So the relative strengths of the United States lie mostly in the easy items. In particular, the easiest PISA items are part of the first pattern, A1. In fact it might be said that pattern A1 represents the strengths of the United States best. It includes almost exclusively easy items, in particular "Speeding fines 01", "Charts 01", "Employment data 01" and "Bike rental 02", which have the highest solution rates for the United States of all the items (from 92.91 % for "Speeding fines 01" to 80.71 % for "Bike rental 02").

Pattern A1: Read data directly from tables and diagrams (one step)

The items in this pattern require students only to understand a short text and read single values directly from a representation provided such as a table or a bar diagram.

Items in this pattern	"Transport 01", "Speeding fines 01", "Charts 01", "Migration 01", "Bike rental 02" and "Employment data 01"
Illustrating items	"Migration 01" and (released) "Charts 01"
Demarcating items	"Computer game 01" and "Running time 01"

The demarcating items, "Computer game 01" and "Running time 01", are two items which at a first glance seem to fit this pattern as well but where the Unites States performed conspicuously poorly in comparison with the OECD average. "Computer game 01" not only represents a weakness of the United States, but in fact shows a significant negative difference when compared to the OECD average and the average of the comparator systems. In order to correctly answer this item, more than one step is needed and the first step is not reading data directly. Rather, students must first calculate total scores and then compare them in a second step. Similarly, in "Running time 01" more than one step is needed to solve this item. Students must first arrange the numbers according to their size and then identify the third-lowest number.

Pattern A2: Simple handling of data from tables and diagrams

The items in this pattern require students to understand a short text, read two values from a given representation and then perform some straightforward operation such as adding or comparing the values.

Items in this pattern	"Population pyramids 01", "Carbon dioxide 02", "Carbon tax 01" and "Employment data 02"
Illustrating item	"Carbon tax 01"
Demarcating items	"Charts 02", "Charts 05" and "Carbon dioxide 03"

The three demarcating items for this pattern, "Charts 02", "Charts 05" and "Carbon Dioxide 03", also seem to fit at first sight. However, the first two of these require more than a straightforward operation. In "Charts 02" a whole development process must be considered over time and "Charts 05" requires the continuation of a trend. "Carbon dioxide 03" is more challenging since students need to handle units (the data in the diagram are given in million metric tonnes) where it is easily possible to make mistakes.

Pattern A3: Handling directly manageable formulae

The items in this pattern require students to use a formula provided, e.g. inserting numbers for variables, and do some easy calculation. The formula can be used directly, without any re-structuring.

Items in this pattern	"Thermometer cricket 01", "Drip rate 03" and "Medicine doses 01"
Illustrating items	"Medicine doses 01" and (released) "Drip rate 03"
Demarcating items	"Carbon tax 02", "Medicine doses 04" and "Drip rate 01"

The demarcating items "Carbon tax 02", "Medicine doses 04" and "Drip rate 01"might seem to fit into this pattern as well since there are formulae involved, but on closer inspection they do not. In "Carbon tax 02" the formula given cannot be used directly because it is necessary to apply some arithmetic laws first ("x before +"). Ignoring this will easily lead to mistakes. In "Medicine doses 04" there are two formulae that have to be compared, the values have to be switched from years to months for the second formula, and the values have to be rounded off appropriately. The item "Drip rate 01" contains a formula that has to be interpreted functionally ("what happens if…") so it does not fit into this pattern either.

Relative weaknesses of the United States

It is remarkable that the patterns which display not only the weaknesses but the relative weaknesses of the United States include nearly all of the most difficult PISA items. The five items with the lowest OECD average solution rates are "Revolving door 02", "Arches 02", "Thermometer cricket 02", "Spacers 03" and "An advertising column" (ranging from 3.5% for "Revolving door 02" up to 8.8% for "An advertising column"). All these items turn up in the following patterns. This suggests that the United States has a particular weakness in the most challenging items.

Pattern B1: Curricular requirement: π is necessary

The items in this pattern require the explicit use of π in a calculation.

Items in this pattern	"Wheelchair basketball 02", "Running tracks 01", "Running tracks 02", "An advertising column 01" and "Revolving door 02" (part of this pattern, but not so significant)
Illustrating item	"Running tracks 01" and (released) "Revolving door 02"
Demarcating item	"Arches 01"

To underline that this pattern is the result of a curricular requirement (being familiar with the number π) we chose "Arches 01" as a demarcating item. As in all other items in this pattern, students have to identify the concrete geometric form of a circle and operate with it in some way. In the item "Arches 01", students have to find the relation of the height (radius) and the width (diameter) of a circular arch. Therefore π is not necessary here**,** although π is used in two of the four possible answers in this multiple-choice item. So it is not circles themselves that seem to be the problem for the U.S. students since they did particularly well in "Arches 01" (see Table 3.6).

Pattern B2: Substantial mathematization of a real world situation

The items in this pattern require students to establish a mathematical model of a given real world situation in the form of a term or an equation with variables for geometric or physical quantities, before further actions (especially calculations) can take place. They have to understand the situation and activate and apply the appropriate mathematical content.

Items in this pattern	"Thermometer cricket 02", "The fence 01", "Arches 02", "Spacers 03" and "Revolving door 02"
Illustrating tem	"Arches 02" and (released) "Revolving door 02"

Pattern B3: Genuine interpretation of real world aspects

The items in this pattern require students to take a given real world situation seriously and genuinely interpret aspects of it. The popular superficial classroom strategy "don't care about the context, just extract the numbers from the text and do some obvious operations" fails here.

Items in this pattern[4]	"Tennis balls 02", "Migration 02", "Migration 03", ("An advertising column 01"), ("Running Tracks 02") and ("Drip rate 01")
Illustrating item	"Migration 02"

Pattern B4: Reasoning in a geometric context

The items in this pattern require genuine reasoning in a planar or spatial geometric context by using geometric concepts and facts. The transformation from the given real world context into the corresponding geometric context is straightforward and not the hard part of the task.

Items in this pattern[4]	"Map 01", "Roof truss design 01", "Revolving door 01", ("Arches 02"), ("Running tracks 01") and ("Running tracks 02")
Illustrating item	"Map 01" and (released) "Revolving door 01"

ANALYSES OF ILLUSTRATING ITEMS

This section illustrates some of the patterns identified in the previous section using relevant publicly released PISA (not all of the patterns can be well illustrated using publicly released items). This should render the patterns more transparent and concrete. As already stated, each single item in the PISA test has its own structure and specific cognitive requirements. A certain pattern is therefore composed of items that share some common features whereas other features are specific to that item. The illustrating items contain these common features, alongside other features that are not typical for the pattern.

Strength patterns (A1 and A3)

Pattern A1: Read data directly from tables and diagrams (one step)

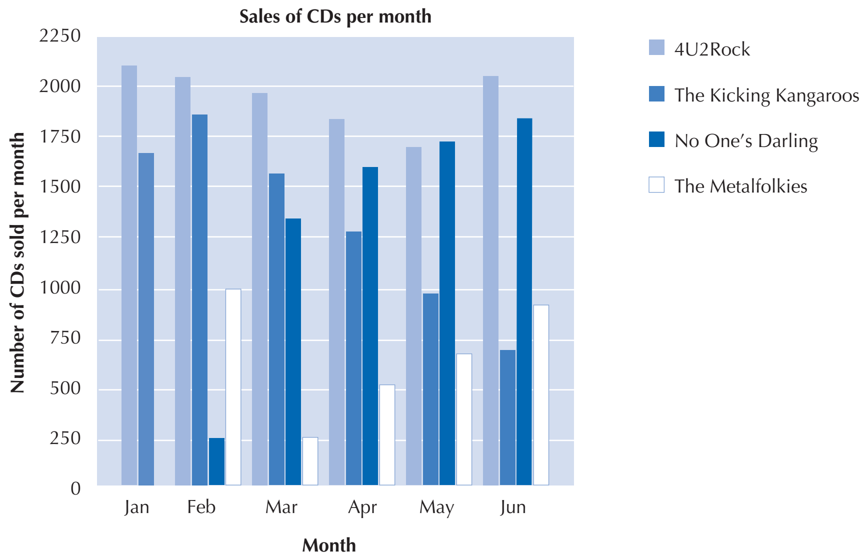
Understanding a short text and reading single values directly from a given representation such as a table or a bar diagram.

"Charts 01"is chosen to illustrate this pattern (Figure 3.3). With its solution rate of 91.93% for the United States, and 87.27% on average for OECD countries it is a rather easy item, but the items "Employment data 01" and "Speeding fines 01" in this pattern have even higher solution rates. In this pattern, only "Transport 01", with a solution rate of 57.82% for the United States and 50.03% for the OECD average, has a much lower solution rate (because of its complex-multiple-choice structure – each individual statement in "Transport 01" is easy). "Charts 01" requires students to interpret given data and belongs to the mathematical content area "Uncertainty and data". This combination of content and process represents a specific strength of the United States students.

■ Figure 3.3. ■

CHARTS 01 (PM918Q01)

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



QUESTION 1

How many CDs did the band *The Metalfolkies* sell in April?

A. 250

B. 500

C. 1 000

D. 1 270

Ideal-typical solution:

In order to answer the question one needs to have a look at the April bars and to identify the fourth bar as representing the number of CDs sold by The Metalfolkies. This bar reaches up to 500.

Therefore the correct answer is: B

The task "Charts 01" is embedded in a real world situation and starts with a short stimulus which explains the structure of the following diagram. The number of CDs sold per month from four different bands is given for the six months from January to June. For each month from February onwards there are four bars. Each bar represents the number of CDs sold in a specific month by a single band. The task for the students in item 01 is to find out how many CDs the band The Metalfolkies sold in April. Therefore one needs to have a look at the white bar for April which represents the band The Metalfolkies. It reaches 500 so answer B is correct.

Pattern A3: Handling directly manageable formulae

Using a given formula directly to perform some easy calculation.

The item "Drip rate 03" can be analyzed to exemplify this pattern. This item has a relatively low solution rate for the United States, at 29.92%, but the OECD average is even lower at 25.72%, so the solution rate is relatively low overall. The items in this pattern are exclusively from the mathematical content area "Change and relationships", and the process of "Drip rate 03" is "Employ". This combination represents another specific strength of the United States students.

■ Figure 3.4. ■

DRIP RATE 03 (PM903Q03)

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate. *D*, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (ml)
v is the volume in ml of the infusion
n is the number of hours the infusion is required to run.

QUESTION 3

Nurses also need to calculate the volume of the infusion, *v*, from the drip rate, *D*.
An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.

What is the volume in ml of the infusion?

Volume of the infusion .... ml

Ideal-typical solution:

First one has to identify the given values and to find out which value is wanted. Then the formula can be converted for v so that afterwards one has only to insert the right values in the new formula and calculate the result. For this task, D is 50 (drops per minute), d is 25 (drops per ml) and n is 3 (hours).

Thus $D = \frac{dv}{60n}$, $v = \frac{D \times 60n}{d} = \frac{50 \times 60 \times 3}{25} = 360\ (ml)$

Therefore the correct answer is: 360.

The stimulus of the "Drip rate" task introduces the topic and explains a formula usedby nurses to calculate the drip rate (in drops per minute) for infusions. This formula contains four variables which are each separately explained in the stimulus. One challenge of the item "Drip rate 03" is to identify the relevant variables and their values. Another requirement is to insert these values correctly into the formula and then calculate the unknown volume of the infusion. A third challenge is to rearrange the formula with respect to the unknown volume of the infusion. This can either be achieved by inserting all given values and then converting the formula, or by converting the formula first and inserting the given values afterwards. The insertion is straightforward and the calculation is easy to perform. The formula is directly manageable since it is pre-structured by the fraction line. The relatively low solution rates in comparison with other items in this pattern are due to the number of algebraic steps required.

Weakness patterns (B1, B2 and B4)

In order to illustrate the patterns for the relative weaknesses of the United States, items 01 and 02 of the unit "Revolving door" were analyzed. Item 02 has a wide range of cognitive requirements and therefore exemplifies several patterns. The following section explains which of the typical features of the patterns are part of the items "Revolving door 01" and "Revolving door 02".

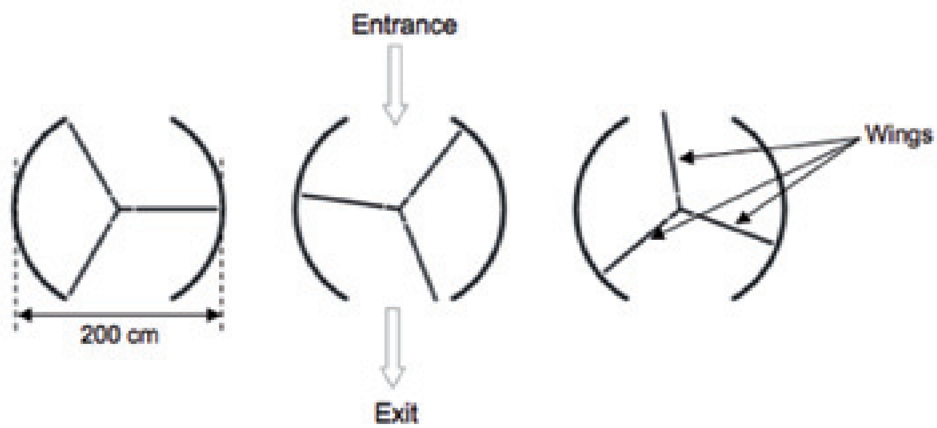
Analysis of the item "Revolving door 01"

The item "Revolving door 01" (Figure 3.5) exemplifies pattern B4 (Reasoning in a geometric context). The United States has a solution rate of 46.96% for this item whereas the OECD average is 57.71%, so this is an item of intermediate difficulty. Pattern B4 has three items which are more difficult ("Arches 02", "Running tracks 01" and "Running tracks 02") but also an easier item ("Roof truss design 01") and one of similar difficulty ("Map 01"). Like all the items in this pattern, the mathematical content area of "Revolving door 01" is "Space and shape", and the process is "Employ".

■ Figure 3.5. ■

REVOLVING DOOR 01 (PM995Q01)

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Question 1: REVOLVING DOOR PM995Q01 – 0 1 9

What is the size in degrees of the angle formed by two door wings?

Size of the angle: ....°

Ideal-typical solution:

The door wings divide the circular-shaped space into three equal sectors; consequently the angle formed by two door wings is $\frac{360^\circ}{3} = 120^\circ$

Therefore the correct answer is: 120°

The stimulus of the item describes a revolving door and presents three diagrams which include information like the diameter and different positions of the door wings. This description already refers to a mathematical model, that is the geometric form of a circle, which is divided into three congruent parts by three linear segments. A difficulty is the rotation of the door wings, so there is no fixed shape. The students are faced with three slightly different diagrams instead of one. Even if the students do not have to mentally revolve the door wings, they need to recognize that the angle between the door wings stay the same when the door is rotated. Such a mental operation with the moving door wings is central for the second item of this task "Revolving door 02" (see below).

The essential cognitive requirement for "Revolving door 01" is to detect that the three sectors of the revolving door are congruent and therefore have the same angles. After that the students have to conclude that one of the angles will be 120°.

For this the students need knowledge about angles, namely that the full angle of a circle comprises 360°. A further difficulty could be that students might be unfamiliar with determining angles in the sectors of a circle. These requirements are mainly features of pattern B4 (Reasoning in a geometric context).

Some mistakes can be expected (as illustrated in the next section). For example, some students will not understand the geometric model and therefore simply pick out the only given value which is the diameter (expected wrong solution 200°). Some students will recognize that they have to divide the full angle by three but do not know how many degrees a full angle comprises or do not have a proper concept of angles, so they might use an avoidance strategy like simply using the value of the diameter for the full angle (expected wrong solution 66. 6°). Another incorrect strategy would be to guess or to measure the angle.

Analysis of the item "Revolving door 02"

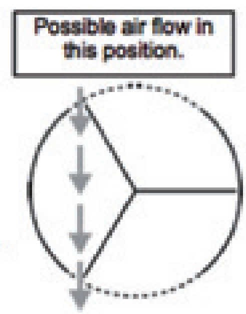
Analyzing the item "Revolving door 02" (Figure 3.6) exemplifies particularly patterns B1 (Curricular requirement: π is necessary) and B2 (Substantial mathematization of a real world situation). "Revolving door 02" has the lowest solution rate on average for the OECD in the whole test. The solution rate for the United States for this item is only 2.26% and the OECD average is 3.47%. The mathematical content area is "Space and shape" and the process is "Formulate". As discussed earlier, this content area represents a particular weakness of the United States.

■ Figure 3.6. ■

REVOLVING DOOR 02 (PM995Q02)

Question 2: REVOLVING DOOR PM995Q02 – 0 1 9

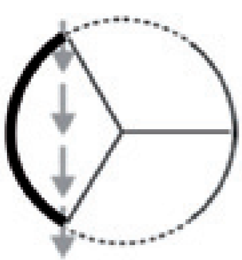
The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



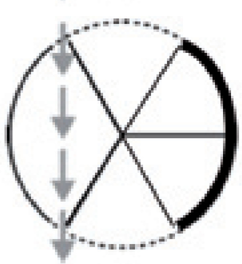
What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .... cm

Ideal-typical solution:



The minimal arc length on the left side of the door has to be one third of the circumference because the sector formed by two door wings is – according to the conditions about air flow – at least one-third of the whole circle.



For reasons of symmetry the minimal arc length on the right side of the door is also one-third of the circumference.

Thus at most one-third of the circumference is left for the two openings, which implies that the maximum opening for exit and entrance is one-sixth of the circumference each:

circumference $= 200 \times \pi \approx 628$ cm   *Arc length* $\approx 628.32 \div 6 \approx 104$ cm

Therefore the correct answer is (rounded off): 104 cm

The stimulus and the first item of this unit have already been analyzed to illustrate pattern B4.

Like “Revolving door 01”, the item “Revolving door 02” describes a real world situation through a text and a diagram. Neither the text nor the diagram in “Revolving door 02” contain the necessary value of the diameter, so the students have to transfer this information from the original stimulus to the new situation given in this item. The diagram leads to a mathematic model, but a substantial difficulty is that the item requires a dynamic conception, because the lengths of the openings are flexible and the rotation of the door wings has to be considered. In particular, in order to identify the arc length of the right side of the circle, it is necessary to modify the diagram and transfer it for a different position of the door wings. Therefore the mathematization of the real world situation is much more difficult than in the first part of the task, “Revolving door 01”. These cognitive requirements are typical for items of pattern B2 (Substantial mathematization of a real world situation). For that reason the item “Revolving door 02“can be used to exemplify this pattern.

It can be expected that a lot of students are not able to formulate an adequate mathematic model of the given situation. Some students might use an improper model, e.g. calculating the length of the arc between two door wings instead of the length of the arc of each opening. Furthermore, the task is unfamiliar and gives students no suggested approaches or instructions. Students have to invent their own strategies. For example, they could consider two extreme cases with the door at the entrance nearly opened and just closed; another possibility is to use a “dividing strategy”, that is to divide the circle into six equal parts and to argue that entrance and exit have to be one sixth of the circumference each.

No matter which strategy they use, students must calculate the circumference of the circle in order to solve the item. For this calculation the students need to know how to use the formula of the circumference (π times diameter or 2π times radius). This requirement is the core of pattern B1 (Curricular requirement: π is necessary) and therefore the item “Revolving door 02” can also be used to exemplify pattern B1.

Students who are not able to use the formula to calculate the circumference can be expected to use empirical strategies to measure the length. For example they may try to measure the length with a ruler or may use the given diameter as a scale.

ANALYSIS OF STUDENT SOLUTIONS

In this section a random sample of 500 student solutions to two of the illustrating items which include important features of the B-patterns are analysed, in order to clarify the results reported so far.

“Revolving door 01”

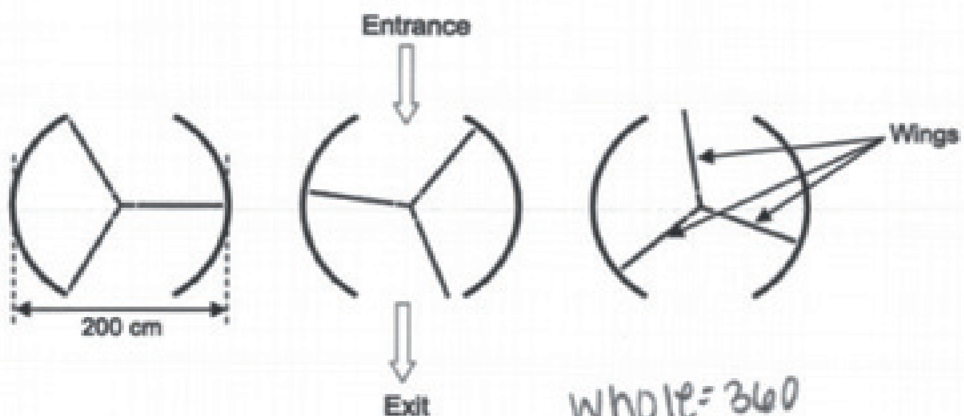
A look at the student solutions of “Revolving door 01” reveals three common mistakes. For most of the wrong answers we cannot interpret the reasons behind the mistakes because most students have only written their answer without indicating a reason for it (they were not required in this item to show the complete calculation).

One of the most common (visible) mistakes is the answer “240”. The students divided the circle correctly into three parts of 120° each and then multiplied one segment by two. The question states “what is the size in degrees of the angle formed by two door wings”. It is possible that these students misunderstood the word “wings” and thus calculated the angle of two segments instead of the angle between two wings. It is also possible that they simply regarded the “two” in the question as a prompt to multiply their first result (120°) by 2 without regarding the geometric situation.

This mistake is shown in the two following solutions and exemplifies pattern B4 (Reasoning in a geometric context).

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular space. The inside diameter of this space is 2 meters (200 centimeters). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



whole= 360
each one= 120
two together= 240

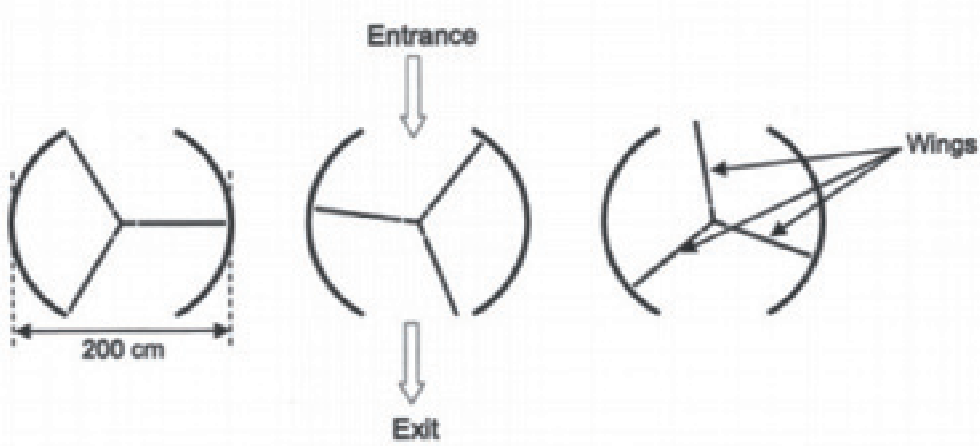
Question 38: REVOLVING DOOR PM995Q01 – 0 1 9

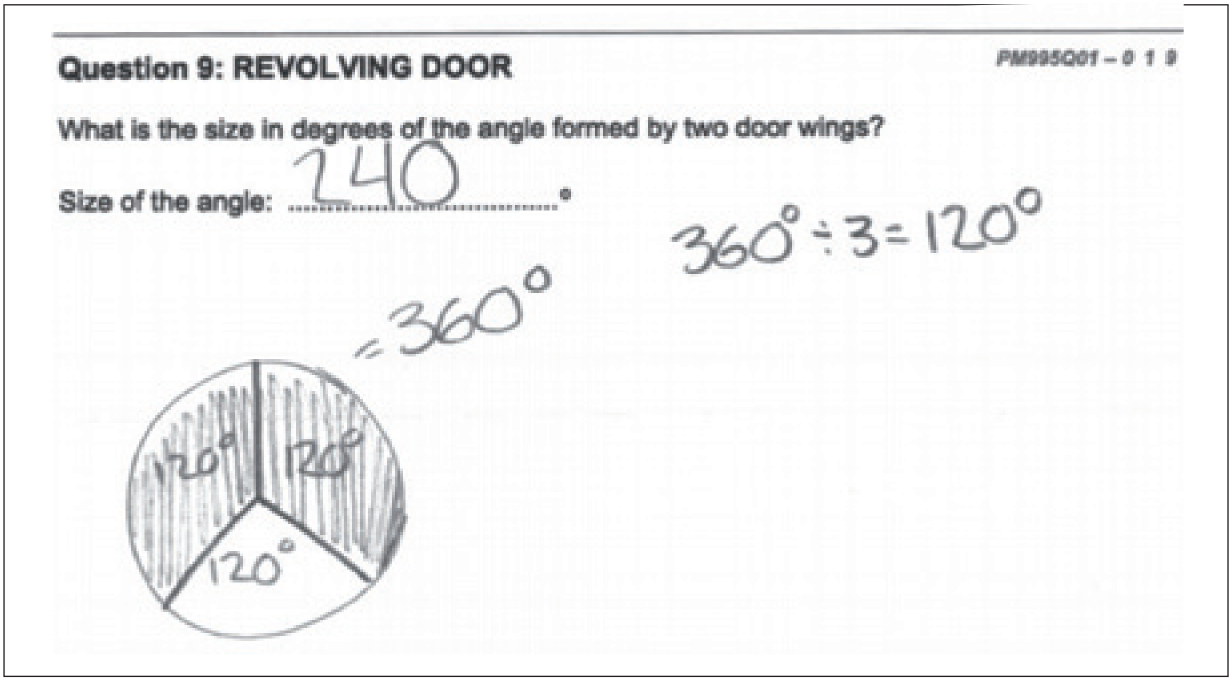
What is the size in degrees of the angle formed by two door wings?

Size of the angle: ....240.....................°

REVOLVING DOOR

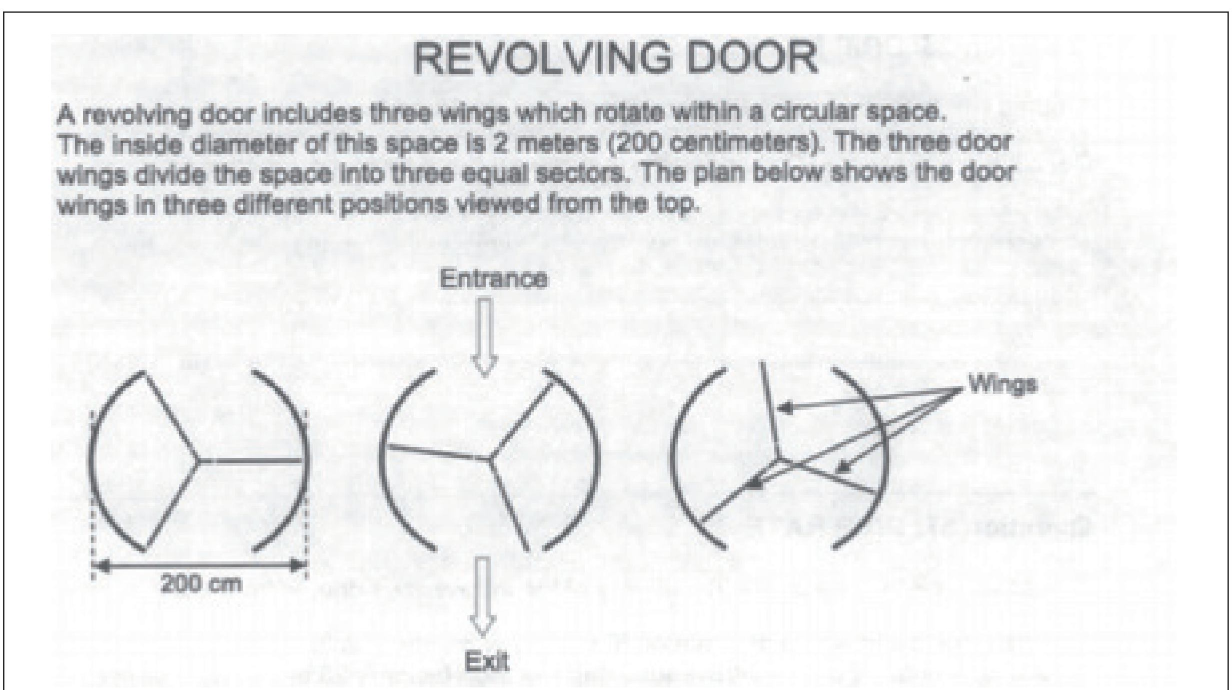
A revolving door includes three wings which rotate within a circular space. The inside diameter of this space is 2 meters (200 centimeters). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.

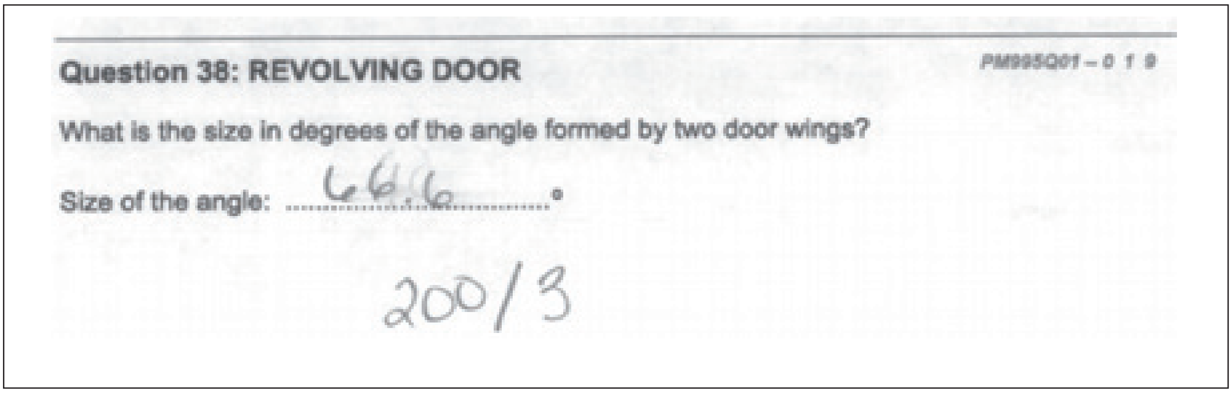




Another common wrong answer is "100". This suggests that these students have flawed knowledge about circles and believe them to have a total angle of 300° or they were simply using the length of the radius as a value for the angle. Instead of reasoning in this geometric situation they simply took the given diameter (200) and divided it by 2 (because of the "two door wings").

The answers "66.7", "66.6" and "67" represent a third common mistake. For these answers we assume that the students simply divided 200 by 3, because 200 cm and (doors) are the numbers given in the text. These students did not reason in the geometric context but simply extracted the number 200 from the stimulus and did the obvious operation of dividing it by three. One of these mistakes is shown in the following solution.




Question 38: REVOLVING DOOR PM995Q01 – 0 1 9

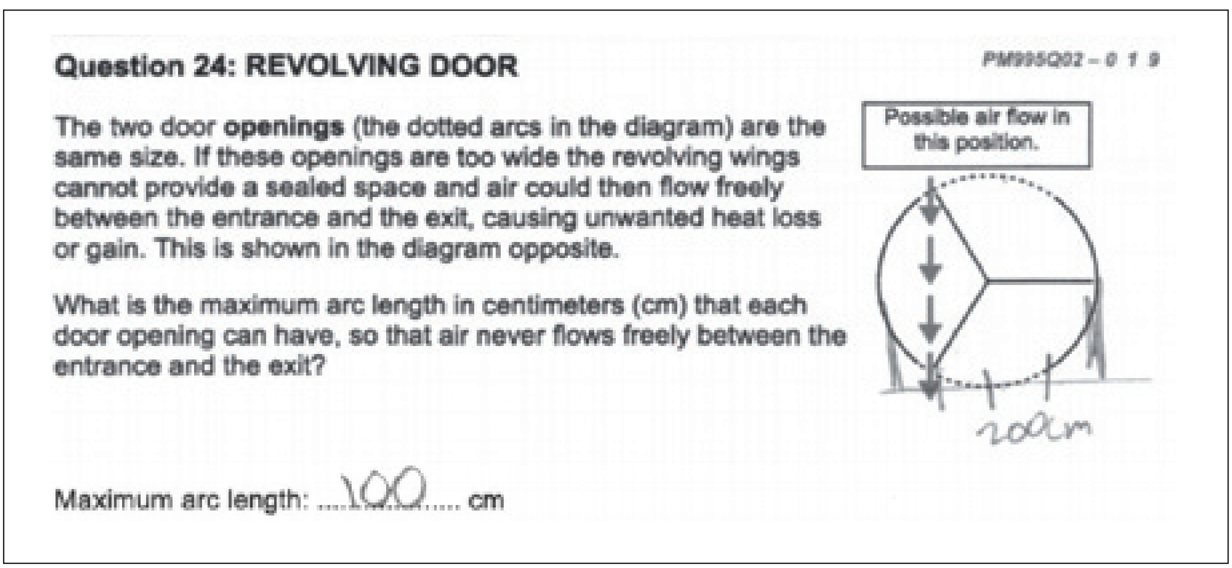
What is the size in degrees of the angle formed by two door wings?

Size of the angle: ......66.6........°

200/3

"Revolving door 02"

For this item, the sample of 500 solutions contains a great variety of answers and also lots of missing answers, which is certainly due to the high level of difficulty of the item. This item also did not require the students to show their working. The most common answer was 100 cm, which is the radius of the circle. Since the diameter is the only given value it can be expected that a lot of students used it to estimate the arc length of one of the openings. An example is the following solution.


Question 24: REVOLVING DOOR PM995Q02 – 0 1 9

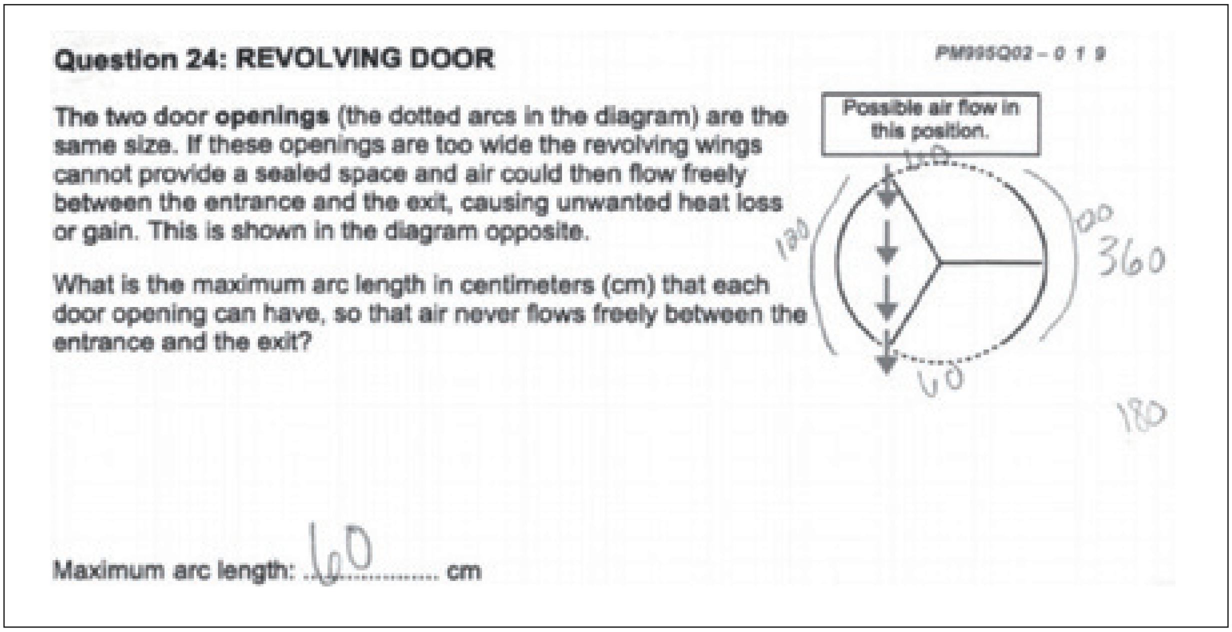
The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimeters (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: ......100...... cm

In this solution the student took the diameter and measured the approximate arc length of the opening. This solution illustrates that the student reconstructed the situation but over-simplified it. He or she tried to figure out the arc length of the opening but ignored the curvature or replaced it by a linear segment. Apparently this student is not able to calculate the circumference.

The next solution also shows that the student was not able to deal with arc lengths. Indeed the solution displays an adequate mathematical model of the situation, and 60° is the correct angle, but the student simply used angles instead of arc lengths.


Question 24: REVOLVING DOOR PM995Q02 – 0 1 9

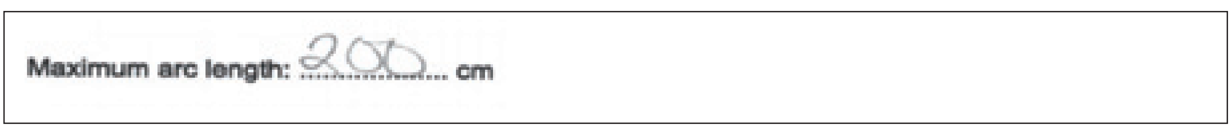
The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimeters (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: ...60........... cm

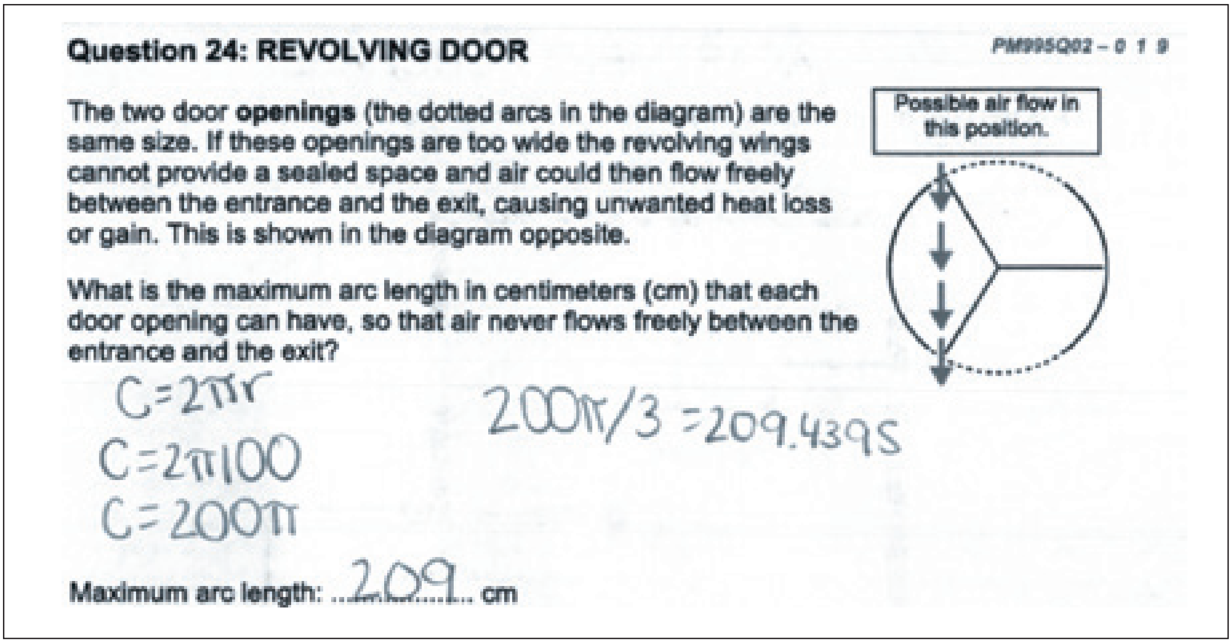
Both solutions therefore exemplify the requirements of pattern B1 (Curricular requirement: π is necessary). The fact that 100 cm and 60 cm were mentioned so often leads to the assumption that a lot of United States students were not able to calculate circumferences. Of course, since most of the students only wrote down the result it is difficult to interpret the underlying mistakes.

The second most common answer was 200 cm, which is the value of the diameter. The students offering this solution just indicated the value 200 cm without further explanations or considerations written down. Probably they did not take the situation seriously and picked out the only given value.


Maximum arc length: ...200... cm

This might be caused by using the superficial classroom strategy "Don't care about the context, just extract the numbers from the text and do some calculation". This solution therefore also fits with pattern B3 (Genuine interpretation of real world aspects).

Nevertheless there are also solutions which display a correct calculation of the circumference. The following two solutions exemplify that some of the United States students have been able to calculate the circumference but had problems with understanding the real world situation or had not been able to formulate an adequate mathematical model. So these two solutions do not fit into pattern B1 (Curricular requirement: π is necessary) but into pattern B2 (Substantial mathematization of a real world problem).



Question 24: REVOLVING DOOR PM995Q02 – 0 1 9

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimeters (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

$C = 2\pi r$

$C = 2\pi 100$

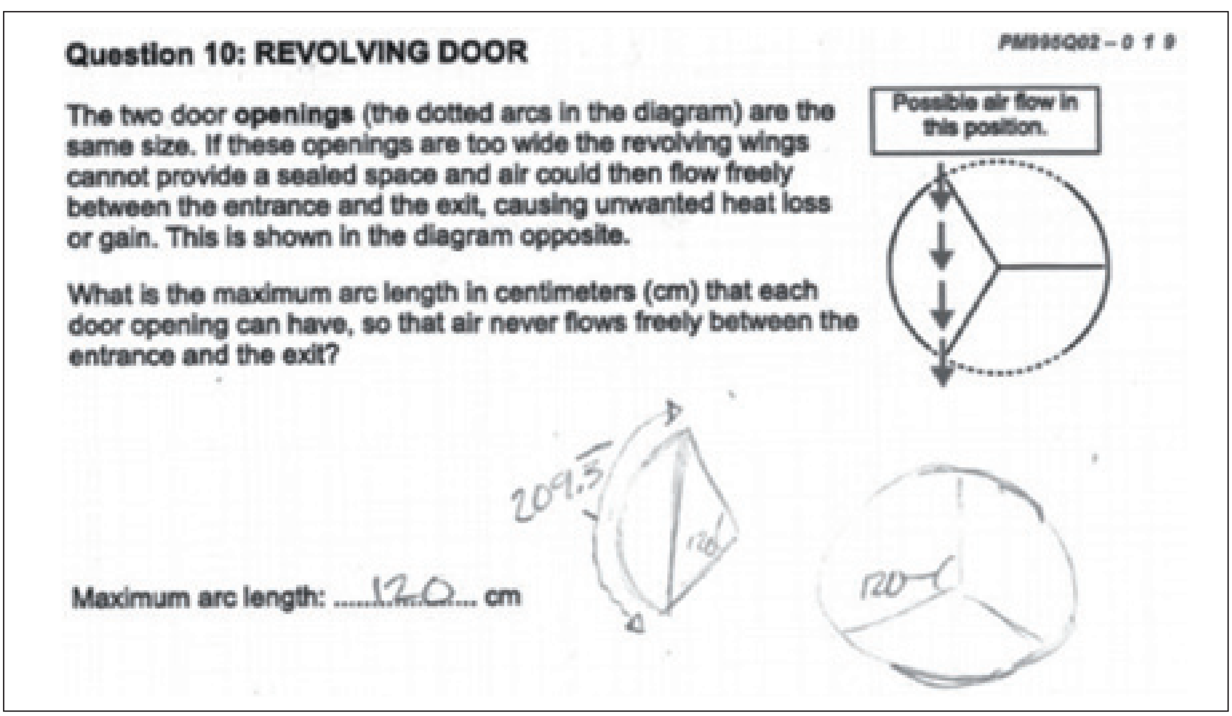
$C = 200\pi$

$200\pi/3 = 209.4395$

Maximum arc length: ....209.... cm

The answer 209 cm was quite frequent. This student first calculated the circumference and then divided it by three. The result is the arc length of one sector. Maybe the student did not understand the situation and therefore used an incorrect mathematical model. But it is also possible that he or she recognized that the length of the opening of entrance and exit in total is one-third of the circumference.

The next student solution is slightly different:



Question 10: REVOLVING DOOR PM995Q02 – 0 1 9

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimeters (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .....120.... cm

Like the solution above, the student calculated one-third of the circumference which is about 209 cm. In contrast to the former solution it is clearly visible that the student calculated the arc length of a sector formed by two door wings and not the total length of the openings. It is noticeable that the student then wrote down the angle of the sector instead of the arc length, so this solution reveals two different student problems. The first problem is the use of the wrong mathematical model, namely considering the arc length of a sector instead of an opening. The second problem seems to be a confusion between angles and arcs.

FINAL OBSERVATIONS

The aim of this chapter was to identify the relative strengths and weaknesses of U.S. students in the PISA 2012 mathematics test. For that purpose, the U.S. solution rates (technically transferred to logits in order to get a linear scale) for all 84 PISA mathematics items that have been administered in the United States are compared with the solution rates of the OECD average and of five reference countries. The reference countries, which have all performed significantly above the OECD average and are, economically speaking, important partners and also competitors of the United States were: Canada, Germany, Korea, the Netherlands and Shanghai-China. Altogether 33 “conspicuous” items were identified, where the differences between the United States and the OECD average, or the five reference countries, were in some way statistically significant. These 33 items revealed certain patterns of relative strengths and weaknesses, where each pattern contains items with similar cognitive requirements. For 16 of these 33 items, the United States performed relatively well, resulting in three strength patterns. For 17 items the United States performed relatively badly, resulting in four weakness patterns.

It is interesting to see that the U.S. students do not perform uniformly compared with the OECD average or with the reference countries. Altogether, the U.S. average (43.8%) in PISA mathematics is below the OECD average (47.5%) and further below the averages of the other five countries. However, there are some items where the U.S. students performed nearly as well as Shanghai-China and better than the OECD average, as well as better than up to four of the other reference countries; these items constitute the relative strengths of the United States. On the other hand, there are items where the United States is much further below the OECD average and the reference countries than the average differences may suggest; these items constitute the relative weaknesses of the United States.

It is remarkable that the strength patterns consist nearly exclusively of easy PISA items and the weakness patterns mostly of demanding items. Both the absolutely easiest and hardest PISA items occur in the patterns, the easiest in the strength and the hardest in the weakness patterns. This is not as obvious as it may seem. It is clear that the United States, like all other countries, performs rather well in easy items and rather moderately in difficult items; this is the definition of “easy” and “difficult”. However, it is not at all obvious that the United States should perform *relatively* well or badly in these items. It seems that the U.S. students have particular strengths in cognitively less-demanding mathematical skills and abilities, such as extracting single values from diagrams or handling well-structured formulae. And they have particular weaknesses in demanding skills and abilities, such as taking real world situations seriously, transferring them into mathematical terms and interpreting mathematical aspects in real world problems. These are tasks where the well-known superficial classroom strategy “Don’t care about the context, just extract the numbers from the text and do some obvious operations” is bound to fail. This strategy is popular all over the world and frequently helps pupils and students to survive in school mathematics and to pass examinations. However, in a typical PISA mathematical literacy task, the students have to use the mathematics they have learned in a well-founded manner. The American students obviously have particular problems with such tasks. Of course, in more demanding tasks, some basic skills such as those mentioned (extracting values or handling formulae) are needed too, so the relative strengths of the U.S. students are a necessary prerequisite for solving higher-order tasks. Therefore, when it comes to the implications of these findings, one clear recommendation would be to focus much more on higher-order activities such as those involved in mathematical modeling (understanding real world situations, transferring them into mathematical models, and interpreting mathematical results), without neglecting the basic skills needed for these activities.

Notes

1. This standard deviation is 0.29, and the average of all differences is -0.21. The resulting threshold values are therefore 0.08 respectively -0.50.

2. This standard deviation is 0.36, and the average is -0.69. The resulting threshold values are therefore -0.33 respectively -1.05.

3. After each round of PISA a number of items are publicly released in order to illustrate what has been assessed in the assessment. These are typically used to illustrate the published framework and the analysis that appears in the international reports from PISA. Not all of the patterns identified in the analysis within Chapter 3 can be illustrated with the publicly released items.

4. Items in parentheses are second category items, which primarily fit in another pattern.

References

Freudenthal, H. (1983), *Didactical Phenomenology of Mathematical Structures*, Reidel, Dordrecht.

OECD (2013), *PISA 2012 Database*.
http://pisa2012.acer.edu.au

OECD (2013), *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*, PISA, OECD Publishing.
http://dx.doi.org/10.1787/9789264190511-en

4

PISA and the U.S. Common Core State Standards for Mathematics

How does PISA relate to the education standards that apply within the United States? Most U.S. states have adopted the Common Core State Standards for Mathematics (CCSSM) as their state mathematics standard. A relevant question therefore is how performance measured by PISA relates to the CCSSM and whether faithful implementation of CCSSM is likely to improve the U.S. performance in the PISA test? This chapter provides an initial investigation into this.

Chapter 2 presented the United States' performance in PISA 2012 and described the factors associated with good performance and high levels of equity. Chapter 3 then analyzed the results at the item level in order to reveal the strengths and weaknesses in the mathematical competencies of 15-year old students in the United States. But how does PISA relate to the education standards that apply within the United States? Most U.S. states have adopted the Common Core State Standards for Mathematics (CCSSM) as their state mathematics standard (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010 – hereafter referred to as CCSSM, 2010). A relevant question therefore is how performance measured by PISA relates to the CCSSM – and would faithful implementation of CCSSM be likely to improve the United States performance in the PISA test? This chapter provides an initial investigation into this by seeking to understand, in mathematical terms, how CCSSM relates to the PISA measures and vice versa.

COMPARING THE PISA ASSESSMENT FRAMEWORK TO THE U.S. COMMON CORE STATE STANDARDS FOR MATHEMATICS

Assessment items can be thought of as samples from some larger domain. Every individual item in PISA is a particular, partial expression of a more general goal, idea, or set of goals and ideas outlined in the PISA framework. No finite set of PISA items quite expresses what is expressed in the PISA framework. Therefore, before relating specific PISA items to CCSSM, it is important to consider the relationship between the PISA framework and CCSSM.

Assessment frameworks and standards documents are designed to serve different purposes. State education standards such as CCSSM are intended to bring coherence to the academic functions of systems for education. If programs for curricula, assessment and instruction are all aligned with the standards, then to that extent they are also aligned with one another and jointly promote the state's vision for learning. Assessment frameworks such as those for PISA, on the other hand, are intended only to specify a measurement construct, not to ground all of the major academic functions of a school system.

Because assessment frameworks and state standards documents are designed to serve different purposes, it is a subtle undertaking to compare one to the other. Nevertheless, to a certain degree, comparisons can be made. That is because each type of document verges to some extent into the other's territory. Education standards aren't designed to specify an assessment program completely, but they do have extremely strong implications for what is assessed. And while assessment frameworks aren't designed to be a basis for an aligned curriculum or instruction, they do make implicit and/or explicit claims about what is valuable for students to learn. A fruitful comparison of the PISA framework and CCSSM can be made if the documents' different purposes are recalled as areas of overlap are explored. Important similarities, and important differences, will emerge from this comparison.

Structure of the Common Core State Standards for Mathematics

In order to understand the procedure used in this chapter for analyzing 2012 PISA items relative to the state standards, it is important to be aware of the high-level structure of the Common Core State Standards for Mathematics. CCSSM includes two distinct kinds of standards: practice standards and content standards. First there are eight Standards for Mathematical Practice, which are as follows:

- MP.1. Make sense of problems and persevere in solving them.
- MP.2. Reason abstractly and quantitatively.
- MP.3. Construct viable arguments and critique the reasoning of others.
- MP.4. Model with mathematics.
- MP.5. Use appropriate tools strategically.
- MP.6. Attend to precision.
- MP.7. Look for and make use of structure.
- MP.8. Look for and express regularity in repeated reasoning.

As described in CCSSM:

> The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual

inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (CCSSM, 2010:6).

As will be described in greater detail later in this chapter, MP.4 ("Model with mathematics") is the most relevant practice standard for a PISA analysis, though others are of some relevance too.

The second kind of standards appearing in CCSSM are the Standards for Mathematical Content. The content standards are presented in two sections, one for kindergarten to eighth grade (K–8) and one for high school. In *Grades K – 8* there are standards for each grade, each of which states an expectation of what students will understand or be able to do by the end of the grade. The content standards for each grade are grouped into clusters and the standards and clusters are in turn organized into domains, such as "Counting and Cardinality" or "Number and Operations – Fractions." Because of the importance of coherence in supporting mathematics achievement, the K – 8 content standards were designed not by a process of sorting topics into grades, but rather by a process of weaving together progressions across grades.

The *High School* content standards are presented in six mathematical categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

For five of these six categories (all except modeling), the content standards are presented using the domain/cluster/standard structure used for grades K–8. In the case of modeling, no dedicated standards are given, but an extended description of modeling is provided, including the modeling cycle discussed later in this chapter (CCSSM 2010, 72-73). In addition, content standards in the other five categories are marked with a star symbol to highlight them as opportunities for modeling.

A closer look at mathematics in the PISA framework

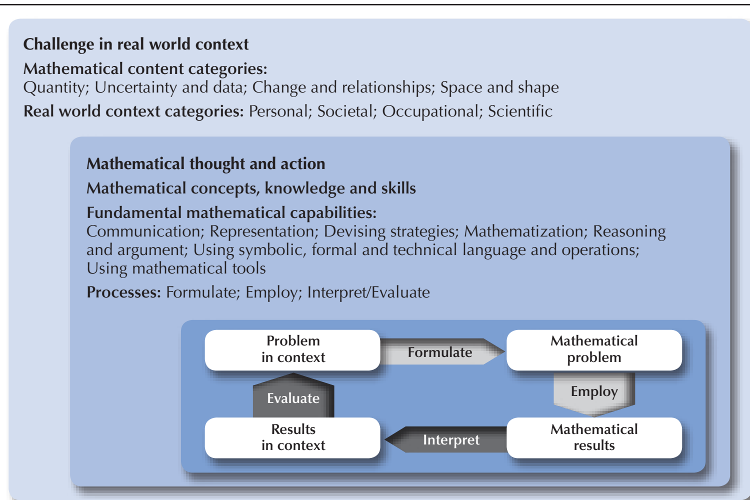
In the PISA framework, mathematical literacy consists of being able to:

1. Formulate situations mathematically
2. Employ mathematical concepts, facts, procedures, and reasoning
3. Interpret, apply and evaluate mathematical outcomes

Formulating, employing, and interpreting have always been the basis for PISA's construct of mathematics. These three capacities are reported as subscales in PISA.

Formulating, employing and interpreting are sequential steps in what the PISA framework refers to as the modeling cycle. See Figure 4.1 below, which is Figure 1.1 in the PISA framework (OECD 2013, p. 26).

■ Figure 4.1 ■
The modeling cycle in the PISA framework



.

PISA items often concentrate on a single process category or step of its modeling cycle. The approximate target distribution of score points by process category is shown in Table 4.1 (see Table 1.1 of OECD 2013, p. 38).

■ Table 4.1. ■
Approximate distribution of PISA items across process categories

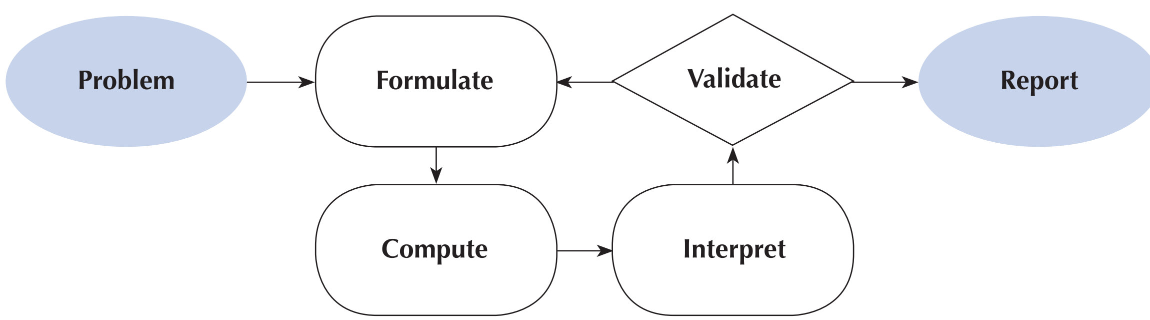
Process category	Percentage of score points
Formulating situations mathematically	Approximately 25
Employing mathematical concepts, facts, procedures and reasoning	Approximately 50
Interpreting, applying and evaluating mathematical outcomes	Approximately 25
TOTAL	**100**

Mathematical literacy in the Common Core State Standards for Mathematics

Mathematical literacy is important in CCSSM. First, from the earliest grades, the standards focus strongly on arithmetic in part because without arithmetic, a person will never be mathematically literate. Second, the phrase "real-world and mathematical problems" is a refrain in the standards (with over 20 occurrences), which points to the standards' balanced attention to pure and applied mathematics. Writings on mathematical literacy appear in the bibliography to the standards. Also themes directly relevant to mathematical literacy appear in follow-up documents to CCSSM such as criteria for publishers produced by the CCSSM writing team.

But the most direct and visible correlate of PISA's construct of mathematical literacy within CCSSM is the high school category of modeling. CCSSM presents a modeling cycle that closely resembles PISA's own (Figure 4.2, from CCSSM 2010, p. 72).

■ Figure 4.2 ■
The Modeling cycle from CCSSM



Note that "Report" in CCSSM is partially related to "Evaluation" in PISA, and also partially related to "Communication", which the PISA framework considers to be an underlying "fundamental mathematical capability", but not explicitly a modeling step or a process category on a par with "Formulate", "Employ" and "Interpret". So there is some relationship between these. And yet, "Report" in CCSSM would appear straightforwardly to suggest that a student might actually be asked to produce a brief piece of writing as part of a modeling exercise and such a step can indeed be found in some modeling tasks present in the field.[1,2] Reporting on this sort of scale is less prevalent in PISA.

■ Table 4.2. ■
Correspondences between the CCSSM Modeling cycle and the PISA Modeling cycle

CCSSM modeling step	Related PISA modeling step
Formulate	Formulate
Compute	Employ
Interpret	Interpret
Validate	Evaluate
Report	N/A

Following through the correspondences in Table 4.2, one can produce a rough map of PISA's approximate target distribution across different aspects of modeling in CCSSM terms (Table 4.3).

■ Table 4.3. ■
Rough map of approximate target distribution of PISA items in CCSSM Modeling terms

CCSSM modeling step	Percentage of related PISA score points
Formulate	Approximately 25
Compute	Approximately 50
Interpret	Approximately 25
Evaluate	Small
Report	Small
TOTAL	**100**

Despite the differences above, what is clear is that in high school, CCSSM – influenced substantially by PISA itself – invests significantly in mathematical literacy, and that this extensive shared territory makes comparisons between PISA and CCSSM both possible and valuable.

ANALYZING THE PISA 2012 MATHEMATICS ITEMS RELATIVE TO CCSSM

Design of the PISA assessment in mathematics

The following analysis is focused on the paper-and-pencil assessment of mathematics in PISA 2012; it does not discuss the computer-based mathematics assessment that PISA implemented in 2012 as a bridge to the 2015 assessment, which will be fully computer delivered.

> In PISA, students encounter the test not as a disconnected sequence of one-off questions, but rather as a series of "units". Each unit presents a description of a real-world context using stimuli such as text, tables, charts and figures. After reading the description of the situation, students answer anywhere from one to four questions about the situation. Thus, an "item" refers to one question in a unit.[3]

For the paper-and-pencil assessment, a total of 56 units containing a total of 110 mathematics items were implemented. However, two clusters of items were solely used in so called "easier" test booklets that were typically used by lower-performing countries. As the United Sates did not use these booklets, the items analyzed in this chapter are restricted to the 84 items that were in the booklets administered in the United States and most other countries.

Issues to consider when analyzing PISA tasks in CCSSM terms

Analyzing PISA tasks in CCSSM terms is a nontrivial exercise. One major source of difficulty is that mathematical literacy – in both PISA and CCSSM – can involve using elementary mathematics to solve sophisticated problems (Steen, 2007). Since the goal of PISA is to assess the cumulative yield of mathematics performance over the school career of 15-year-olds, rather than solely what students have learned at the age of 15 or at 10th grade level, the mathematical techniques needed for some PISA tasks might be first introduced in CCSSM during the middle grades, or perhaps even the elementary grades. Nevertheless, it would be wrong to call such a task an elementary-grade task according to CCSSM. That is because PISA tasks typically require problem-solving processes that are not expected in CCSSM at these grades, and that very few elementary school students could command in any case. A valid procedure of analysis ought to capture accurately the "lag" between content and process in tasks without misleadingly suggesting either that PISA is too easy or that CCSSM is too hard.

A second complication arises from the fact that the Standards for Mathematical Practice are not a taxonomy. Traditional procedures for coding content should not be applied to the Standards for Mathematical Practice. A mathematical task, or a mathematical behavior, might easily exhibit/evoke/resonate with several practice standards at once – or none. Thus, in a collection of *N* items, there is no reason why the sum of the weights across the eight practice standards must add up to *N*. Practice standards cannot be thought of even approximately as drawers in a filing cabinet.

Because mathematical literacy in PISA relates so closely to modeling in CCSSM, one expects that every PISA task will involve practice standard MP.4. MP.4 tells a large part of the story of how PISA and CCSSM relate through the lens of the practice standards, though the other standards for mathematical practice are of some relevance also.

Data generated for each analyzed task

In order to address the complications described above and analyze the PISA tasks relative to CCSSM in the most appropriate way, an analysis procedure to generate the following data for each of the 84 PISA 2012 mathematics items that were administered in the United States:

1. Progression co-ordinate
2. Modeling attributes (includes the use of technology and tools)
3. Modeling intensity level

These terms are defined next and illustrated using publicly released items from the PISA 2012 assessment.

Progression co-ordinate

Viewed from a high enough level, the CCSSM content standards could be described as a sequence of time-ordered sets of content goals.

The first set in the sequence is the kindergarten content standards; the last set is the high school content standards.

If one considers that the CCSSM document D consists of, or defines, a sequence of time-ordered sets of content goals $D_1, \ldots, D_n,$ one refers to the subscript k in D_k as the "progression co-ordinate". Progression co-ordinates for CCSSM are defined in Table 4.4.

■ Table 4.4. ■
Definition of progression co-ordinates for CCSSM

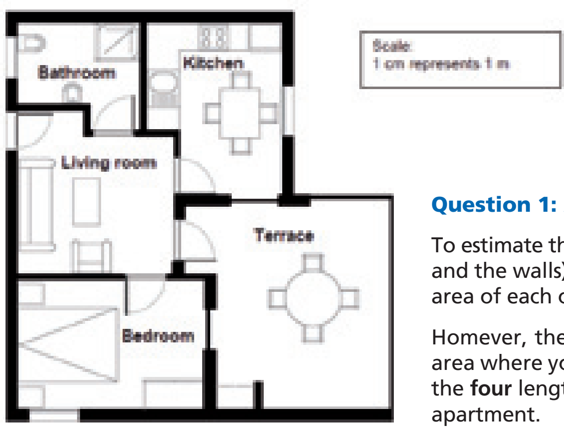
Progression co-ordinate	Content goal set
K	D_k
0	Standards for Mathematical Content—Kindergarten
1	Standards for Mathematical Content—Grade 1
2	Standards for Mathematical Content— Grade 2
3	Standards for Mathematical Content— Grade 3
4	Standards for Mathematical Content— Grade 4
5	Standards for Mathematical Content— Grade 5
6	Standards for Mathematical Content— Grade 6
7	Standards for Mathematical Content— Grade 7
8	Standards for Mathematical Content— Grade 8
9	Standards for Mathematical Content—High School

Mathematical tasks, such as PISA items, can be assigned a progression co-ordinate by reference to a theoretically optimal user of content at that grade level, i.e. one who can reliably complete all tasks up to that level. Thus, just because a task has progression co-ordinate k, this does not mean that CCSSM sets an expectation that all students at the corresponding grade level be able to reliably complete the task. The most one can say in general is that a task with progression co-ordinate k is at or beyond the corresponding grade level.

As an illustration of the analysis of progression co-ordinate, we consider the following PISA 2012 released mathematics item (Figure 4.3), one of the 84 items analyzed in this chapter.

■ Figure 4.3. ■
Apartment purchase (Item PM00FQ01)

This is the plan of the apartment that George's parents want to purchase from a real estate agency.



Question 1: APARTMENT PURCHASE

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together.

Homever, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the **four** lengths that are needed to estimate the total floor area of the apartment.

This item was assigned the progression co-ordinate $k = 4$ in light of content standard 4.MD.7d. This standard reads as follows (emphasis added):

> Recognize area as additive. **Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems** (CCSSM 2010, p. 25)

The processes required for success in the "Apartment purchase" task depend on, but are not reducible to, the knowledge and skills that are articulated in content standard 4.MD.7d. Therefore, just because the "Apartment purchase" task has content progression $k = 4$, this does not mean that the PISA test includes fourth-grade tasks, nor does it mean that CCSSM requires fourth graders to solve tasks appropriate for 15-year-olds.

Modeling attributes

Each PISA item was located along five continua that reflect various attributes of modeling tasks (Box 4.1). In each of the following continua, the left-hand side is "less about modeling" while the right-hand side is "more about modeling.[4]"For each of the five attributes, the task was located along the continuum using heuristic values 0, 1, 2.

Box 4.1 **Attributes of modeling tasks**

A_1. Well posedness
Well posed → element(s) of intentional ambiguity → freedom to specify and simplify the problem.
Model is judged correct/incorrect → model is judged useful/not useful, a good start or not.

A_2. Authenticity
Using specified math the real goal → life and realism the real goal, with math content a means not an end.
Cooked-up data/facts → actual or realistic data/facts.
Problem posed with words → problem posed with words and artifacts.
Units abstracted away → units or dimensional thinking prominent in problem and solution.

A_3. Scaffolding
Routine task → non-routine task.
Fully scaffolded → partly scaffolded → unscaffolded.
Key variables are declared or evident → key variables not declared/not evident.
Easy to identify the math to use → careful thought about quantities and their relationships required in order to identify the math to use.

A_4. Coverage of the Modeling cycle
Less coverage → more coverage (formulate, compute, interpret, critique/validate, improve, report).

A_5. Technology and tools
No technology or tools required → technology or tools required → required without prompting.

Note that "Technology and tools" includes technology such as spreadsheets, calculators, graphing software, and so on, as well as tools such as measurement scales or even remembered formulas.

As an illustration of the analysis of modeling attributes, consider the following publicly released PISA 2012 mathematics item, again one of the 84 items analyzed in this chapter. Attributes of the item are shown in Table 4.5.

■ Figure 4.4. ■
MP3 Players (Item PM904Q02)

Translation Note: The use of zeds is important to the unit, so please do not adapt "zed" into an existing currency.

Question 2: MP3 PLAYERS

Olivia added the prices for the MP3 player, the headphones and the speakers on her calculator.

The answer she got was 248.

Olivia's answer is incorrect. She made one of the following errors. Which error did she make?

A. She added one of the prices in twice.
B. She forgot to include one of the three prices.
C. She left off the last digit in one of the prices.
D. She subtracted one of the prices instead of adding it.

■ Table 4.5. ■
Modeling attributes of the "MP3 Players" (Item PM904Q02)

Attribute	Relevant continua	Value assigned (0, 1, 2)	Comment
A_1. Well-posedness	Well posed → element(s) of intentional ambiguity → freedom to specify and simplify the problem Model is judged correct/incorrect → model is judged useful/not useful, a good start or not	0	The task is completely well posed. There are no meaningful choices to be made by the student in representing the situation mathematically. There is an unambiguous notion of correctness for the task. The mathematical work will not be seen as part of any iterative process of refinement.
A_2. Authenticity[5]	Using specified math the real goal → life and realism the real goal, with math content a means not an end Cooked-up data/facts → actual or realistic data/facts Problem posed with words → problem posed with words and artifacts Units abstracted away → units or dimensional thinking prominent in problem and solution	1	It is clear that life is the real goal of the task. The problem is posed with words and artifacts. However, the numbers in the problem are clearly devised in such a way as to foster particular strategies. The quantities in the problem are dimensioned (units of zeds), but units are not prominent in the problem or solution.

Attribute	Relevant continua	Value Assigned (0, 1, 2)	Comment
A_3. Scaffolding	Routine task → non-routine task Fully scaffolded → partly scaffolded → unscaffolded Key variables are declared or evident → key variables not declared/not evident Easy to identify the math to use → careful thought about quantities and their relationships required in order to identify the math to use	1	The task is not entirely routine. But the multiple-choice format implies a certain amount of scaffolding. The universe of possible errors is presented and constricted to a small set, rather than having likely errors defined by the student.
A_4. Coverage of the Modeling cycle	Less coverage → more coverage (formulate, compute, interpret, critique/ validate, improve, report)	0	The answer choices can be seen as different potential models for what went wrong in Olivia's calculation. But the fact that the models appear as multiple-choice options weakens this sense, and it would be a subtle reading of the task in any case. A student would not likely understand his or her work on the task in these terms.
A_5. Technology and tools	No technology or tools required → technology or tools required → required without prompting	1	A calculator can be helpful in completing the task; its use is not prompted.

As this example shows, critical judgment is involved in assigning a number 0, 1, or 2 to each attribute, which required some subjective judgment. Therefore, the specific ratings given should not be over-interpreted.

Modeling level

Each PISA item was also assigned an overall modeling level (Box 4.2):[6]

Box 4.2 **Modeling levels**

Level 0 Pure mathematics, no context

Proof … fluency … procedure … concept … single or multi-step

Level 1 Modeling/application[7]

1.1 One-step problem using D_k content … problem with "thin context"[8]
1.2 Multi-step problem; lower values along most or all of the modeling dimensions A_1– A_5

Level 2 Modeling

2.1 Well-posed multi-step problem; higher values along at least some of the modeling dimensions A_1– A_5
2.2 Less than well-posed problems with model formulation; computation; and interpretation of results
2.3 Least well-posed problems and/or four or more steps of the modeling cycle

For example, the "Apartment purchases" task and the "MP3 players" items were both assigned Level 1.2.

The term "multi-step" should be interpreted judiciously. The "steps" in a "multi-step" item might not take the form of a sequence of discrete calculations; in some cases, the "steps" might instead take the form of multiple nodes in a web of reasoning. Conversely, consider a problem about substituting a value into a complicated equation. Arriving at the final numerical answer might involve several sub-calculations, but such a problem could well be thought of as "single-step" anyway.

Modeling levels were not designed to "spread out" the PISA tasks. They were designed to reflect the full spectrum of tasks that one might see in a faithful implementation of CCSSM.

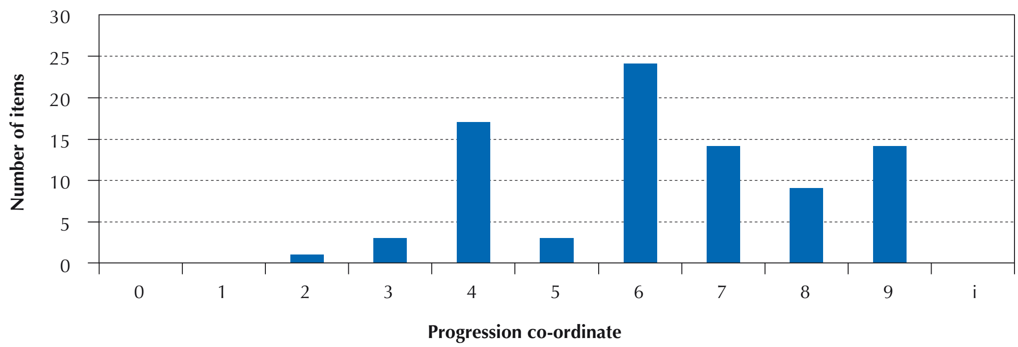
RESULTS OF ANALYZING PISA 2012 ITEMS RELATIVE TO THE U.S. CCSSM

Progression co-ordinate

Mathematical literacy has been described as involving the application of mathematics to solve sophisticated problems (Steen, 2007). Given PISA's intent to assess the cumulative yield of mathematics performance up to the age of 15 years, rather than solely to assess the mathematical content acquired at the age of 15 years, then one would expect PISA items to vary in their progression co-ordinates. Figure 4.5 shows the relative frequency of the 84 items administered in the United States by their progression co-ordinates.

■ Figure 4.5. ■

Distribution of PISA items by progression co-ordinate



The maximum of the distribution is $k = 6$. This is consistent with PISA's aims, reflecting the fact that at grade 6, the progressions in CCSSM lead to the development of new and powerful tools such as ratio, rate, and percent; new ideas such as center and variation; and the culmination of decimal computation.

Fewer items' progression co-ordinates fall at $k = 8$; when they do, they are more often related to the CCSSM Functions domain than to the Expressions and Equations domain.

There is also a noticeable spike in the distribution at grade 4. One of the main factors leading to this peak is that grade 4 is when CCSSM first expects students to be using all four basic operations to solve multi-step problems with whole numbers, including interpreting remainders (4.OA.3). In light of what PISA tasks are aiming to do, the peak in this distribution at grade 4 reveals the fact that the skills referenced in 4.OA.3 already represent quite a powerful body of mathematics. Standard 4.OA.3 deserves to be thought of right alongside 7.EE.3 as a "pinnacle". (Indeed, 4.OA.3 might be thought of as a "whole-number version" of 7.EE.3, which is concerned with solving multi-step real-life and mathematical problems posed with positive and negative rational numbers.)

A secondary factor contributing to the spike at grade 4 is that the numerical values in PISA items may be simple enough to foster mental computations. (All other things being equal, the two calculations 120/6 and 139.13/7.662 would receive progression co-ordinates of 4 and 6, respectively.)

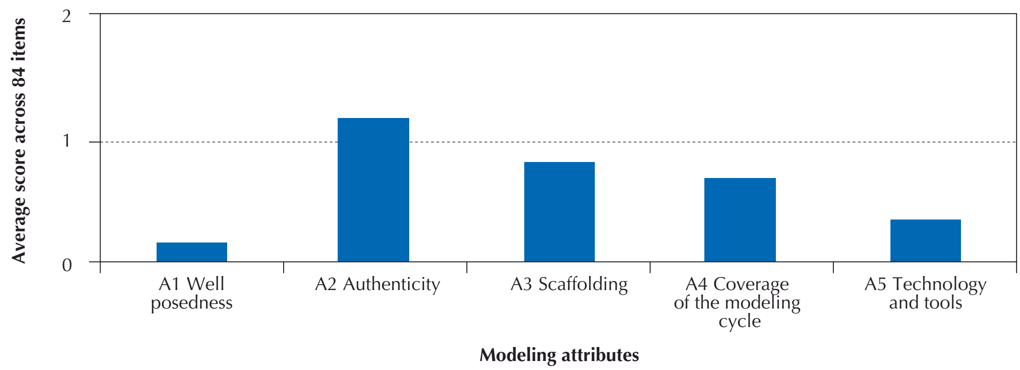
Steen's observation (Steen, 2007) about humble mathematics being put to sophisticated uses is often thought of specifically in relation to arithmetic, algebra and geometry. But it is equally true when applied to statistics. Only four of the PISA 2012 items with progression co-ordinates of 9 were referable to the high school Statistics and Probability category in CCSSM – about the same number of items at that level referable to the high school Algebra category (three items) or the high school Functions category (four items). This too is consistent with PISA's purposes. Statistics, as it is used in work and life, relies little on advanced techniques of the kind first introduced in high school. It relies, rather, on fertile uses of the fundamental notions of center, variation, and distribution that are first introduced in the middle grades in CCSSM and that continue to be relevant throughout high school, college, work and life.

Modeling attributes

Figure 4.6 shows the average score in each modeling attribute. That is, for each modeling attribute, the 0-1-2 scores for that attribute were summed over all 84 items and then divided by 84. A brief discussion of each attribute follows the figure.

■ Figure 4.6. ■

Average score by modeling attribute



Attribute A_1 *(Well posedness).* Problems that are anything less than perfectly posed have always been rare on standardized tests – in fact, usually such problems are completely absent by design. Reasons for this include psychometric difficulties as well as the difficulty of explaining the ins and outs of modeling to a public unused to seeing such work on tests. Against this backdrop, the 18 PISA problems from 2012 that scored above zero on this attribute represent a qualitatively greater proportion than one would typically find in the item bank for a U.S. state mathematics test from before CCSSM. This reflects PISA's intent to assess the capacity of students to identify problems even in ambiguous contexts, as they would be in real-world situations. At the same time, it should be noted that none of those 18 items scored a 2 on this attribute, and many are more well-posed than some prototypical modeling tasks one finds in the field.[9]

Attribute A_2 *(Authenticity).* PISA 2012 items had relatively high authenticity, as is clear enough from a brief scan of the released items. PISA items are often posed using interesting artifacts from real life, they often involve units and dimensional thinking, and they often put mathematics in service of the phenomenon.

Attribute A_3 *(Scaffolding).* PISA includes a mixture of completely unscaffolded items, partly scaffolded items, and highly scaffolded items. In some cases, a multiple-choice answer format effectively serves intentionally or unintentionally to scaffold an item. Psychometric considerations and the cost of hand scoring have a strong effect on the degree of scaffolding in tasks.

Attribute A_4 *(Coverage of the modeling cycle).* The average score for this attribute was relatively low. This was to be expected, as the design of PISA is such that individual items tend to focus on a single step of the cycle.

Attribute A_5 *(Technology and tools).* Many PISA items that involve computations do not actually require a calculator, as the chosen numbers are amenable to mental arithmetic, or the rubric may allow an approach that involves rounding to convenient numbers. Where technology and tools are in play, the technology involved is generally only a four-function calculator. Of course, it is not be feasible to include spreadsheets and other tools central to working quantitatively in the paper-and-pencil test. Tasks in the computer assessment component of PISA will lend themselves to a higher core on attribute A_5.

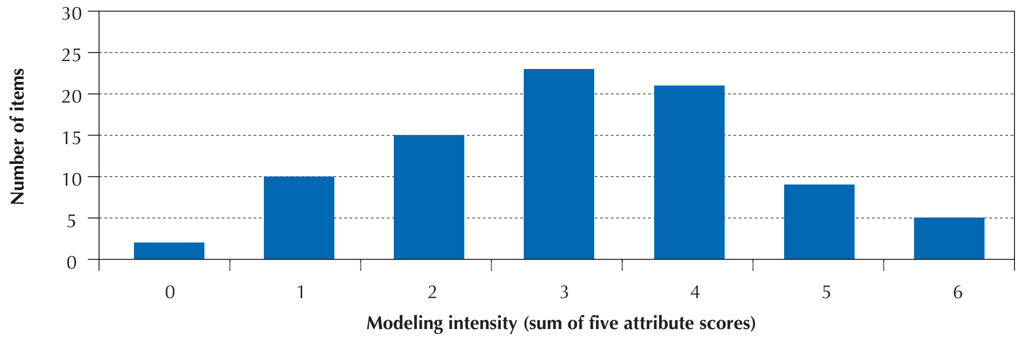
Modeling intensity

A heuristic measure of *modeling intensity* can be defined by summing the 0-1-2 attribute scores across all five modeling attributes. If this is done, the 2012 PISA items exhibit the distribution shown in Figure 4.7, which covers data for the 84 items administered in the United States.

The highest attribute sum for any of the 84 items was 6. Items with an intensity of 6 tended to be the "richest" of the modeling exercises – and also among the most difficult, as will be discussed further below.

No pronounced relationship was found between tasks' progression co-ordinates and their modeling intensities.

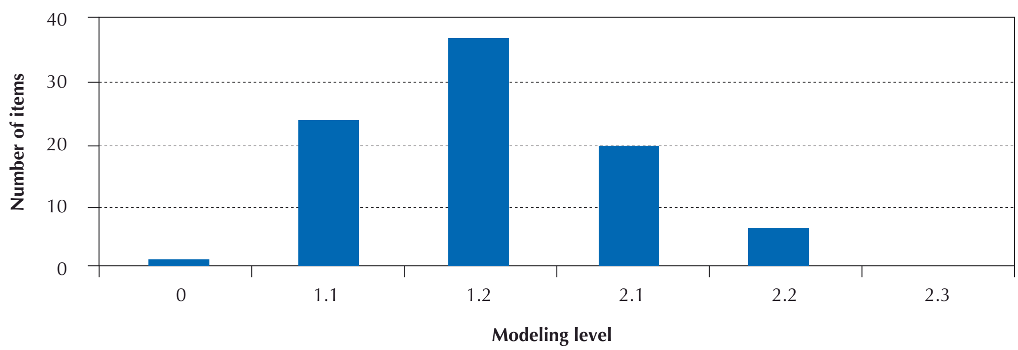
■ Figure 4.7. ■
Distribution of PISA items by modeling intensity



Modeling Levels

Most of the PISA items cluster at level 1.2 and its two neighbor levels, as shown in Figure 4.8, which reports data only for the 84 items administered in the United States.

■ Figure 4.8. ■
Distribution of PISA items by modeling level



This distribution likely represents a higher general level of modeling than has been present in U.S. mathematics tests prior to CCSSM. For a number of reasons, including the nature of the construct being measured, items on United States state mathematics tests prior to CCSSM would likely cluster at lower levels (namely 0, 1.1, and a smaller amount of 1.2, with 2.1 also present in some cases).

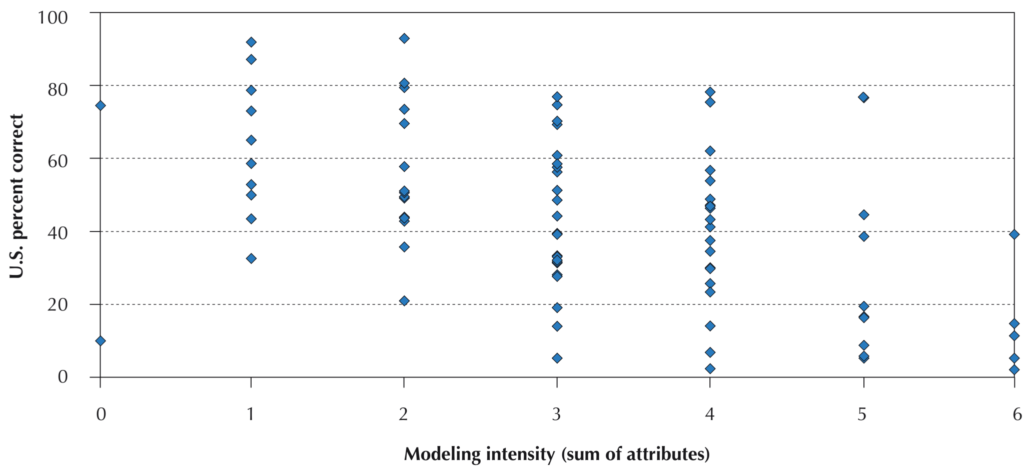
Table 4.6 provides the full detail of the classification of the 84 PISA 2012 mathematics items against the dimensions of the CCSSM.

Relationship to student performance

It is interesting to see how students in the United States performed according to modeling intensities (sum of attribute scores). Figure 4.9 clearly shows a relationship between modeling intensity and performance (percent correct on the item). Additionally, Figure 4.10 shows the comparison between performance and "content plus practices" by simply adding the progression co-ordinate to the modeling intensity. Again a distinct relationship is evident, with R^2 values of around 0.3.

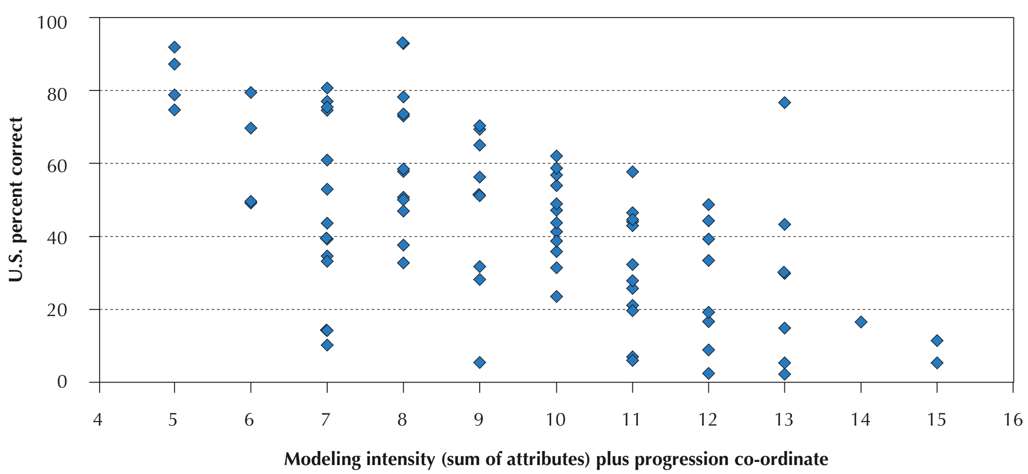
■ Figure 4.9. ■

U.S. percent correct decreases with increasing modeling intensity



■ Figure 4.10. ■

U.S. percent correct decreases with increasing sum of modeling intensity and progression co-ordinate



FINAL OBSERVATIONS

The analysis in this chapter has shown some commonality between the PISA framework and items and the CCSSM. It classifies PISA 2012 items against the CCSSM progression according to where they sit in the progression of standards up to high school level, the degree to which they represent attributes of modeling and their modeling level. This has served to show a degree of commonality between the PISA and CCSSM constructs. This chapter began by asking whether faithfully implementing CCSSM would improve PISA results. The analysis suggests that this is intuitively plausible. The prominence of modeling in the U.S. high school standards has already influenced developers of large-scale assessment in the United States It will take time for excellent task designs to emerge and become widespread. But it seems safe to say that over the coming years, the high school curriculum in the United States will attend to modeling to a greater degree than has happened in the past. If more students work on more and better modeling tasks than happens today, then one could reasonably expect PISA performance to improve.

It may be that U.S. students seldom work on well-crafted tasks that situate algebra, proportional relationships and rational numbers within authentic contexts. More generally, one wonders if the application problems that most students have access to today are the worst of all worlds: fake applications that strive to make the mathematics curriculum more palatable, yet do no justice either to modeling or to the pure mathematics involved.

In effect, it could be argued that when it comes to authentic modeling, there has been too little "opportunity to learn". If, as a result of quality implementation of CCSSM, the U.S. curriculum begins presenting high school students with better modeling problems – and if assessments included them as well – then it would be reasonable to believe, in a way that goes beyond simple correlations, that PISA scores would improve. That would be important not because of any drive for higher test scores *per se*, but rather because it would begin to signal that the United States is becoming a country whose citizens make frequent and productive use of mathematics in their work and life. What is certain, from an examination of the 2012 PISA 2012 items and our performance on them, is that the United States is not that country today.

■ Table 4.6 ■
Mapping of PISA items to aspects of the Common Core State Standards for Mathematics

Name	Item code	Progression co-ordinate k	A1 Well posedness	A2 Authenticity	A3 Scaffolding	A4 Coverage of the modeling cycle	A5 Technology and tools	Level of modeling intensity	Attribute Sum
Apartment Purchase	PM00FQ01	4	0	2	2	0	0	1.2	4
An Advertising Column	PM00GQ01	9	0	2	2	1	1	2.1	6
Wheelchair Basketball	PM00KQ02	7	0	1	1	1	1	1.2	4
A View with a Room	PM033Q01	7	0	1	0	0	0	1.2	1
Bricks	PM034Q01	7	0	0	1	0	0	1.2	1
Population Pyramids	PM155Q01	6	0	2	1	0	0	1.1	3
Population Pyramids	PM155Q02	6	0	2	1	0	1	1.2	4
Population Pyramids	PM155Q03	7	1	2	2	0	1	2.1	6
Population Pyramids	PM155Q04	6	0	2	2	0	0	2.1	4
Containers	PM192Q01	8	0	1	0	1	0	1.2	2
Pipelines	PM273Q01	4	0	0	1	0	1	1.2	2
Map	PM305Q01	7	0	2	0	0	2	1.2	4
Running Tracks	PM406Q01	7	1	1	1	1	1	2.1	5
Running Tracks	PM406Q02	7	1	1	1	1	1	2.1	5
Lotteries	PM408Q01	9	0	2	0	1	0	2.1	3
Diving	PM411Q01	6	0	2	1	0	1	1.2	4
Diving	PM411Q02	6	0	2	1	1	0	1.2	4
Transport	PM420Q01	6	1	1	0	0	0	1.1	2
Tossing Coins	PM423Q01	7	0	0	0	0	0	1.1	0
Braille	PM442Q02	3	1	2	0	1	0	1.2	4
The Thermometer Cricket	PM446Q01	2	0	2	0	1	0	1.1	3
The Thermometer Cricket	PM446Q02	8	0	2	1	1	1	2.1	5
Tile Arrangement	PM447Q01	8	0	0	1	0	0	1.1	1
The Third Side	PM462Q01	7	0	0	0	0	0	0.0	0
The Fence	PM464Q01	4	0	1	2	0	0	1.2	3
Running Time	PM474Q01	4	0	1	0	0	1	1.2	2
Cash Withdrawal	PM496Q01	4	0	1	0	1	0	1.2	2
Cash Withdrawal	PM496Q02	4	0	1	1	1	0	1.2	3
Telephone Rates	PM559Q01	6	0	1	2	1	0	2.1	4
Chair Lift	PM564Q01	6	1	1	1	1	0	2.2	4
Chair Lift	PM564Q02	6	1	1	2	1	1	2.2	6
Stop the Car	PM571Q01	9	0	1	0	1	0	1.2	2
Number Check	PM603Q01	4	0	1	1	1	0	1.2	3
Number Check	PM603Q02	4	0	1	1	1	1	1.2	4
Computer Game	PM800Q01	4	0	0	1	0	1	1.2	2
Labels	PM803Q01	7	1	1	1	1	0	2.2	4
Carbon Dioxide	PM828Q01	9	1	2	1	0	0	2.2	4
Carbon Dioxide	PM828Q02	9	0	1	0	0	0	1.1	1
Carbon Dioxide	PM828Q03	9	0	1	0	0	1	1.1	2
Drip Rate	PM903Q01	9	0	1	1	1	0	1.1	3
Drip Rate	PM903Q03	9	0	1	1	1	1	1.2	4
Tennis Balls	PM905Q01	4	0	2	1	1	0	1.2	4
Tennis Balls	PM905Q02	5	0	1	2	1	1	1.2	5
Crazy Ants	PM906Q01	6	0	1	0	0	1	1.1	2
Crazy Ants	PM906Q02	9	0	0	0	1	1	1.2	2
Speeding Fines	PM909Q01	6	0	1	0	1	0	1.1	2
Speeding Fines	PM909Q02	6	0	1	1	1	0	1.2	3
Speeding Fines	PM909Q03	6	0	1	1	1	0	1.2	3
Carbon Tax	PM915Q01	6	0	1	0	0	0	1.2	1
Carbon Tax	PM915Q02	6	0	0	0	1	0	1.2	1
Charts	PM918Q01	4	0	1	0	0	0	1.1	1
Charts	PM918Q02	4	0	1	1	1	0	1.2	3
Charts	PM918Q05	8	1	1	1	1	1	2.2	5
Z's Fan Merchandise	PM919Q01	4	0	1	0	0	0	1.2	1
Z's Fan Merchandise	PM919Q02	4	0	1	1	1	0	1.2	3
Sailing Ships	PM923Q01	7	0	1	0	0	0	1.1	1
Sailing Ships	PM923Q03	8	0	0	0	1	1	1.2	2
Sailing Ships	PM923Q04	9	0	2	2	1	1	2.1	6

Name	Item code	Progression co-ordinate k	A1 Well posedness	A2 Authenticity	A3 Scaffolding	A4 Coverage of the modeling cycle	A5 Technology and tools	Level of modeling intensity	Attribute Sum
Sauce	PM924Q02	6	0	1	1	1	0	1.2	3
Arches	PM943Q01	7	0	0	1	1	0	1.1	2
Arches	PM943Q02	8	0	1	2	1	0	2.1	4
Roof Truss Design	PM949Q01	8	0	1	1	1	0	1.1	3
Roof Truss Design	PM949Q02	8	0	1	1	1	0	1.1	3
Roof Truss Design	PM949Q03	7	0	1	1	1	0	1.1	3
Flu Test	PM953Q02	9	0	2	0	1	0	2.1	3
Flu Test	PM953Q03	9	0	1	1	1	0	1.2	3
Flu Test	PM953Q04	9	1	1	1	1	1	2.1	5
Medicine Doses	PM954Q01	6	0	1	1	1	0	1.1	3
Medicine Doses	PM954Q02	6	0	1	1	1	0	2.1	3
Medicine Doses	PM954Q04	6	0	1	1	1	1	2.1	4
Migration	PM955Q01	6	0	2	0	0	0	1.1	2
Migration	PM955Q02	6	1	2	1	1	0	1.1	5
Migration	PM955Q03	6	0	2	1	1	1	1.1	5
Employment Data	PM982Q01	4	0	1	0	0	0	1.1	1
Employment Data	PM982Q02	4	0	1	0	1	1	1.1	3
Employment Data	PM982Q03	5	0	1	0	1	1	1.2	3
Employment Data	PM982Q04	9	1	1	1	1	0	2.2	4
Spacers	PM992Q01	3	0	2	1	1	0	2.1	4
Spacers	PM992Q02	3	0	2	1	1	0	2.1	4
Spacers	PM992Q03	6	0	1	1	1	0	2.1	3
Revolving Door	PM995Q01	4	0	1	2	1	0	2.1	4
Revolving Door	PM995Q02	7	0	2	2	1	1	2.1	6
Revolving Door	PM995Q03	6	0	2	2	1	0	2.1	5
Bike Rental	PM998Q02	5	0	1	0	1	0	1.1	2
Bike Rental	PM998Q04	8	0	1	1	1	0	1.2	3
London Eye	PM934Q01	7	0	1	0	1	0	1.2	2
London Eye	PM934Q02	7	0	0	1	1	0	1.2	2
Seats in a Theatre	PM936Q01	8	0	1	0	1	0	1.1	2
Seats in a Theatre	PM936Q02	8	0	1	1	1	0	1.1	3
Racing	PM939Q01	9	0	0	1	1	0	1.2	2
Racing	PM939Q02	9	0	0	1	1	0	1.2	2
Climbing Mt. Fuji	PM942Q01	6	1	1	1	1	1	1.2	5
Climbing Mt. Fuji	PM942Q02	7	0	2	2	1	1	2.1	6
Climbing Mt. Fuji	PM942Q03	6	0	1	1	1	1	2.1	4
Part-Time Work	PM948Q01	2	0	2	1	0	0	2.1	3
Part-Time Work	PM948Q02	6	1	1	0	1	0	1.1	3
Part-Time Work	PM948Q03	7	0	2	2	1	1	2.1	6
Helen the Cyclist	PM957Q01	6	0	0	1	1	0	1.1	2
Helen the Cyclist	PM957Q02	9	0	1	0	1	0	1.2	2
Helen the Cyclist	PM957Q03	9	0	1	1	1	0	1.2	3
Chocolate	PM961Q02	6	1	1	2	1	1	2.2	6
Chocolate	PM961Q03	9	0	2	1	1	0	2.1	4
Chocolate	PM961Q05	6	1	1	1	2	1	2.2	6
Wooden Train Set	PM967Q01	5	0	2	2	1	0	1.1	5
Wooden Train Set	PM967Q03	5	0	1	1	1	0	1.1	3
Which Car?	PM985Q01	4	0	1	1	1	0	1.2	3
Which Car?	PM985Q02	5	0	1	0	0	0	1.1	1
Which Car?	PM985Q03	6	1	1	0	0	1	1.1	3
Garage	PM991Q01	i	0	0	0	0	0	1.2	0
Garage	PM991Q02	9	0	1	2	1	1	2.1	5

Notes

1. See for example *http://www.comap.com/product/textbooks/*, and the "Karnataka" task (in original form due to COMAP) on slide 21 of *http://www.parcconline.org/sites/parcc/files/GA_CCSS.pdf*

2. See for example "The Taxicab Problem" on p.74 of the Smarter Balanced content specifications (in original form due to the Shell Centre), *www.smarterbalanced.org/wordpress/wp-content/uploads/2011/12/Math-Content-Specifications.pdf*

3. Although PISA items are grouped into units, items are treated as independent in the PISA scoring model. Thus, for example, units are designed so that the answer to a later item in the unit does not depend upon having answered an earlier item in the unit correctly.

4. No value judgments are being made about the relative worth of tasks from either end of the continua. Neither extreme suffices in itself to meet CCSSM as written.

5. Of course, there is such a thing as a *mathematically* authentic task as well. Using the word to describe modeling here is not intended to compete with any of the word'*s other uses.*

6. Again, no value judgments are being made about the relative worth of tasks at each level or sub-level. Neither extreme suffices to meet CCSSM as written.

7. Note, all Level 1 problems are well-posed.

8. For an example of "thin context", see Illustrative Mathematics (undated) *http://www.illustrativemathematics.org/illustrations/436*

9. See for example *www.mathmodels.org/problems/probview.php?probnum=20053*

References

Illustrative Mathematics (undated), A-SSE Animal Populations, Illustrative Mathematics website, *www.illustrativemathematics.org/illustrations/436*.

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010), *Common Core State Standards for Mathematics*, National Governors Association Center for Best Practices, Council of Chief State School Officers: Washington, D.C. *www.corestandards.org/assets/CCSSI_Math%20Standards.pdf*

OECD (2013), *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*, PISA, OECD Publishing, Paris.
http://dx.doi.org/10.1787/9789264190511-en.

Steen (2007), *Moving Beyond Standards and Tests* p.4: ["High schools focus on elementary applications of advanced mathematics whereas most people really make more use of sophisticated applications of elementary mathematics. … Many who master high school mathematics cannot think clearly about percentages or ratios."]
www.stolaf.edu/people/steen/Papers/07carnegie.pdf

Annex A1

THE PISA APPROACH TO ASSESSING STUDENT PERFORMANCE IN MATHEMATICS

THE PISA APPROACH TO ASSESSING STUDENT PERFORMANCE IN MATHEMATICS

The PISA definition of mathematical literacy

The focus of the PISA 2012 assessment of mathematics was on measuring an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals in recognizing the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

The definition asserts the importance of mathematics for full participation in society and it stipulates that this importance arises from the way in which mathematics can be used to describe, explain and predict phenomena of many types. The resulting insight into phenomena is the basis for informed decision making and judgments.

Literacy in mathematics described in this way is not an attribute that an individual has or does not have; rather, it can be acquired to a greater or lesser extent, and it is required in varying degrees in society. PISA seeks to measure not just the extent to which students can reproduce mathematical content knowledge, but also how well they can extrapolate from what they know and apply their knowledge of mathematics, in both new and unfamiliar situations. This is a reflection of modern societies and workplaces, which value success not by what people know, but by what people can do with what they know.

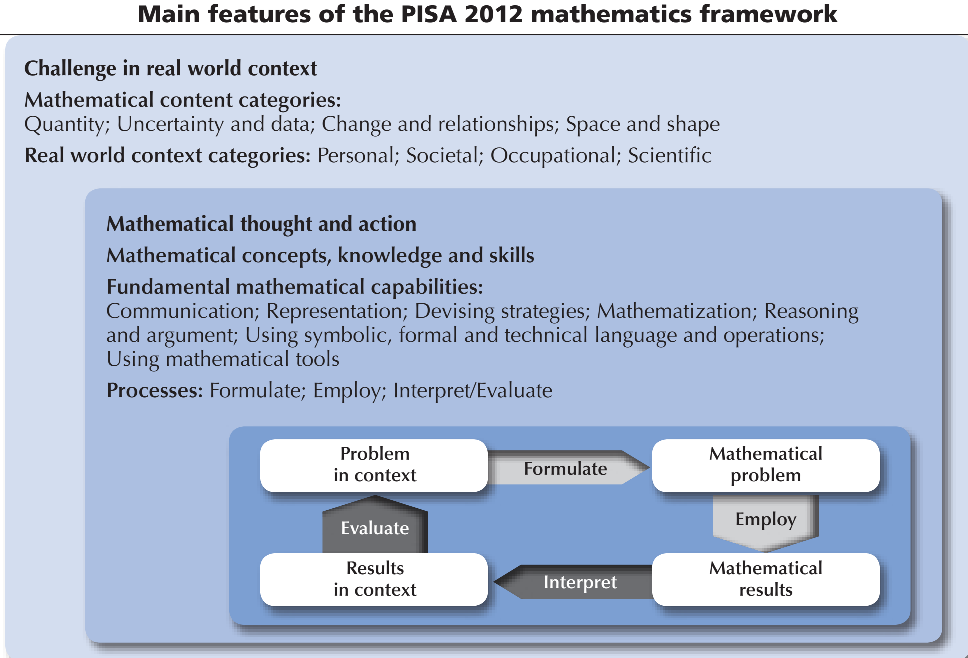
The focus on real-life contexts is also reflected in the reference to using "tools" that appears in the PISA 2012 definition of mathematical literacy. The word "tools" here refers to physical and digital equipment, software and calculation devices that have become ubiquitous in 21st century workplaces. Examples for this assessment include a ruler, a calculator, a spreadsheet, an online currency converter and specific mathematics software, such as dynamic geometry. Using these tools require a degree of mathematical reasoning that the PISA assessment is well-equipped to measure.

The PISA 2012 framework for assessing mathematics

Figure A1.1 presents an overview of the main constructs of the PISA 2012 mathematics framework that was established and agreed by the participating countries, and how the constructs relate to each other. The largest box shows that mathematical literacy is assessed in the context of a challenge or problem that arises in the real world. The middle box highlights the nature of mathematical thought and action that can be used to solve the problem. The smallest box describes the processes that the problem solver uses to construct a solution.

■ Figure A1.1 ■

Main features of the PISA 2012 mathematics framework



Context categories

Real-world challenges or situations are categorized in two ways: their context and the domain of mathematics involved. The four context categories identify the broad areas of life in which the problems may arise: personal, which is related to individuals' and families' daily lives; societal, which is related to the community – local, national or global – in which an individual lives; occupational, which is related to the world of work; or scientific, which is related to the use of mathematics in science and technology. According to the framework, these four categories are represented by equal numbers of items.

Content categories

As seen in Figure A1.1, the PISA items also reflect four categories of mathematical content that are related to the problems posed. The four content categories are represented by approximately equal proportions of items. For the assessment of 15-year-olds, age-appropriate content was developed.

The content category *quantity* incorporates the quantification of attributes of objects, relationships, situations, and entities in the world, which requires an understanding of various representations of those quantifications, and judging interpretations and arguments based on quantity. It involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns, and employing number sense, multiple representations of numbers, mental calculation, estimation, and assessment of reasonableness of results.

The content category *uncertainty and data* covers two closely related sets of issues: how to identify and summarize the messages that are embedded in sets of data presented in different ways, and how to appreciate the likely impact of the variability that is inherent in many real processes. Uncertainty is part of scientific predictions, poll results, weather forecasts and economic models; variation occurs in manufacturing processes, test scores and survey findings; and chance is part of many recreational activities that individuals enjoy. Probability and statistics, taught as part of mathematics, address these issues.

The content category *change and relationships* focuses on the multitude of temporary and permanent relationships among objects and circumstances, where changes occur within systems of interrelated objects or in circumstances where the elements influence one another. Some of these changes occur over time; some are related to changes in other objects or quantities. Being more literate in this content category involves understanding fundamental types of change and recognizing when change occurs so that suitable mathematical models can be employed to describe and predict change.

The content category *space and shape* encompasses a wide range of phenomena that are encountered everywhere: patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, navigation, and dynamic interaction with real shapes and their representations. Geometry is essential to space and shape, but the category extends beyond traditional geometry in content, meaning and method, drawing on elements of other mathematical areas, such as spatial visualization, measurement and algebra. Mathematical literacy in *space and shape* involves understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes.

Process categories

The smallest box of Figure A1.1 shows a schema of the stages through which a problem-solver may move when solving PISA tasks. The action begins with the "problem in context". The problem-solver tries to identify the mathematics relevant to the problem situation, formulates the situation mathematically according to the concepts and relationships identified, and makes assumptions to simplify the situation. The problem-solver thus transforms the "problem in context" into a "mathematical problem" that can be solved using mathematics. The downward-pointing arrow in Figure A1.1 represents the work undertaken as the problem-solver employs mathematical concepts, facts, procedures and reasoning to obtain the "mathematical results". This stage usually involves mathematical manipulation, transformation and computation, with and without tools. The "mathematical results" then need to be interpreted in terms of the original problem to obtain the "results in context". The problem-solver thus must interpret, apply and evaluate mathematical outcomes and their reasonableness in the context of a real-world problem. The three processes – formulate, employ and interpret – each draw on fundamental mathematical capabilities, which, in turn, draw on the problem-solver's detailed mathematical knowledge.

However, not all PISA tasks engage students in every stage of the modeling cycle. Items are classified according to the dominant process and results are reported by these processes, formally named as:

- Formulating situations mathematically
- Employing mathematical concepts, facts, procedures and reasoning
- Interpreting, applying and evaluating mathematical outcomes

Fundamental mathematical capabilities

Through a decade of experience in developing PISA items and analyzing the ways in which students respond to them, a set of fundamental mathematical capabilities has been established that underpins performance in mathematics. These cognitive capabilities can be learned by individuals in order to understand and engage with the world in a mathematical way. Since the PISA 2003 framework was written, researchers (e.g. Turner, 2013) have examined the extent to which the difficulty of a PISA item can be understood, and even predicted, from how each of the fundamental mathematical capabilities is used to solve the item. Four levels describe the ways in which each of the capabilities is used, from simple to complex. For example, an item involving a low level of communication would be simple to read and require only a simple response (e.g. a word); an item involving a high level of communication might require the student to assemble information from various different sources to understand the problem, and the student might have to write a response that explains several steps of thinking through a problem. This research has resulted in sharper definitions of the fundamental mathematical capabilities at each of four levels. A composite score has been shown to be a strong predictor of PISA item difficulty. These fundamental mathematical capabilities are evident across the content categories, and are used to varying degrees in each of the three mathematical processes used in the reporting. The PISA framework (OECD, 2013c) describes this in detail.

The seven fundamental mathematical capabilities used in the PISA 2012 assessment are described as follows:

Communication is both receptive and expressive. Reading, decoding and interpreting statements, questions, tasks or objects enables the individual to form a mental model of the situation. Later, the problem-solver may need to present or explain the solution.

Mathematizing involves moving between the real world and the mathematical world. It has two parts: formulating and interpreting. Formulating a problem as a mathematical problem can include structuring, conceptualizing, making assumptions and/or constructing a model. Interpreting involves determining whether and how the results of mathematical work are related to the original problem and judging their adequacy. It directly relates to the *formulate* and *interpret* processes of the framework.

Representation entails selecting, interpreting, translating between and using a variety of representations to capture a situation, interact with a problem, or present one's work. The representations referred to include graphs, tables, diagrams, pictures, equations, formulae, textual descriptions and concrete materials.

Reasoning and argument is required throughout the different stages and activities associated with mathematical literacy. This capability involves thought processes rooted in logic that explore and link problem elements so as to be able to make inferences from them, check a justification that is given, or provide a justification of statements or solutions to problems.

Devising strategies for solving problems is characterized as selecting or devising a plan or strategy to use mathematics to solve problems arising from a task or context, and guiding and monitoring its implementation. It involves seeking links between diverse data presented so that the information can be combined to reach a solution efficiently.

Using symbolic, formal and technical language and operations involves understanding, interpreting, manipulating and making use of symbolic and arithmetic expressions and operations, using formal constructs based on definitions, rules and formal systems, and using algorithms with these entities.

Using mathematical tools involves knowing about and being able to use various tools (physical or digital) that may assist mathematical activity, and knowing about the limitations of such tools. The optional computer-based component of the PISA 2012 mathematics assessment has expanded the opportunities for students to demonstrate their ability to use mathematical tools.

Paper-based and computer-based media

PISA 2012 supplemented the paper-based assessment with an optional computer-based assessment, in which specially designed PISA units were presented on a computer and students responded on the computer. Thirty-two of the 65 participating countries and economies took part in this computer-based assessment. For these countries and economies, results are reported for the paper-based assessment scale and supplemented with a computer-based scale and a combined paper-and-computer scale ((see Annex B3 in OECD, 2013).

The design of the computer-based assessment ensures that mathematical reasoning and processes take precedence over mastery of using the computer as a tool. Each computer-based item involves three aspects:

- the mathematical demand (as for paper-based items);
- the general knowledge and skills related to information and communication technologies (ICT) that are required (e.g. using keyboard and mouse, and knowing common conventions, such as arrows to move forward). These are intentionally kept to a minimum; and
- competencies related to the interaction of mathematics and ICT, such as making a pie chart from data using a simple "wizard", or planning and implementing a sorting strategy to locate and collect desired data in a spreadsheet.

Response types

The response types distinguish between selected response items and constructed response items. Selected response items include simple multiple choice, complex multiple choice, in which students must select correct answers to a series of multiple-choice items, and, for computer-based items, "selected response variations", such as selecting from options in a drop-down box. Constructed response items include those that can be scored routinely (such as a single number or simple phrase, or, for computer-based items, those for which the response can be captured and processed automatically), and others that need expert scoring (e.g. responses that include an explanation or a long calculation).

References

OECD (2013), *PISA 2012 Results : What Students Know and Can Do – Student Performance in Mathematics, Reading and Science (Volume I),* PISA, OECD Publishing.
http://dx.doi/10.1787/978964201118-en

Turner, R., J. Dossey, W. Blum and **M. Niss** (2013), "Using mathematical competencies to predict item difficulty in PISA", in M. Prenzel, M. Kobarg, K. Schöps and S. Rönnebeck (eds.), *Research on PISA: Research Outcomes of the PISA Research Conference 2009,* Dordrecht, Springer, pp. 23-37.

■ Table A1.1 [Part 1/2] ■

Items by solution rates for United States and comparator countries

		Solution rates (percentage of students providing the correct answer)							Framework categories	
Name	Item Code	United States	OECD average	Canada	Germany	Netherlands	Korea	Shanghai-China	Content	Process
Apartment purchase	PM00FQ01	37.60	44.67	51.25	42.78	59.47	62.02	70.48	Space and shape	Formulate
An advertising column	PM00GQ01	5.37	8.78	9.24	11.26	15.47	15.13	18.00	Space and shape	Formulate
Wheelchair basketball	PM00KQ02	7.01	14.86	19.15	21.26	17.80	14.57	45.83	Space and shape	Formulate
A View Room	PM033Q01	73.04	75.79	77.84	81.60	80.25	78.22	78.73	Space and shape	Interpret
Bricks	PM034Q01T	32.69	42.40	48.07	48.17	46.07	55.57	54.04	Space and shape	Formulate
Population Pyramids	PM155Q01	69.39	67.68	74.31	69.58	79.92	77.51	72.66	Change and relationships	Interpret
Population Pyramids	PM155Q02D	62.09	61.59	73.20	69.57	79.55	73.49	76.15	Change and relationships	Employ
Population Pyramids	PM155Q03D	14.82	18.69	25.63	22.07	29.22	34.08	42.97	Change and relationships	Employ
Population Pyramids	PM155Q04T	53.92	55.88	63.73	58.62	51.02	64.44	69.20	Change and relationships	Interpret
Containers	PM192Q01T	35.86	42.45	50.22	50.77	59.48	58.18	64.88	Change and relationships	Formulate
Pipelines	PM273Q01T	49.22	51.48	53.41	57.02	62.21	64.14	73.40	Space and shape	Employ
Map	PM305Q01	46.45	60.37	63.47	66.34	67.05	62.46	71.95	Space and shape	Employ
Running Tracks	PM406Q01	16.69	25.63	35.52	30.90	29.79	40.12	57.83	Space and shape	Employ
Running Tracks	PM406Q02	8.87	16.90	24.32	19.40	22.91	30.06	45.13	Space and shape	Formulate
Lotteries	PM408Q01T	33.40	39.41	47.91	49.43	36.86	39.02	53.74	Uncertainty and data	Interpret
Diving	PM411Q01	47.16	51.11	58.31	55.60	63.10	61.24	70.77	Quantity	Employ
Diving	PM411Q02	48.94	45.73	51.92	49.66	62.04	56.45	71.11	Uncertainty and data	Interpret
Transport	PM420Q01T	57.82	50.03	64.12	50.70	57.67	28.19	60.67	Uncertainty and data	Interpret
Tossing Coins	PM423Q01	74.59	79.06	80.05	81.24	83.79	81.91	94.43	Uncertainty and data	Interpret
Braille	PM442Q02	34.66	38.28	43.11	42.09	39.28	56.06	58.58	Quantity	Interpret
Thermometer Cricket	PM446Q01	74.74	68.59	77.93	65.00	73.19	79.33	88.07	Change and relationships	Formulate
Thermometer Cricket	PM446Q02	5.38	6.82	8.87	7.24	13.73	16.22	41.05	Change and relationships	Formulate
Tile Arrangement	PM447Q01	65.06	68.36	73.37	74.17	79.16	85.13	83.35	Space and shape	Employ
Third Side	PM462Q01D	10.16	12.20	7.62	12.49	7.32	44.38	76.82	Space and shape	Employ
The Fence	PM464Q01T	14.08	23.67	30.46	32.58	23.51	41.98	59.82	Space and shape	Formulate
Running Time	PM474Q01	69.64	74.33	74.61	77.50	75.19	81.77	80.75	Quantity	Employ
Cash Withdrawal	PM496Q01T	49.60	52.99	59.87	55.42	58.05	63.96	70.98	Quantity	Formulate
Cash Withdrawal	PM496Q02	60.92	66.76	70.43	69.97	68.77	80.00	82.53	Quantity	Employ
Telephone Rates	PM559Q01	56.81	63.16	65.55	66.75	67.53	81.35	88.22	Quantity	Interpret
Chair Lift	PM564Q01	41.28	46.12	48.48	51.56	57.97	63.37	71.27	Quantity	Formulate
Chair Lift	PM564Q02	39.30	45.82	51.86	48.39	58.58	52.25	74.74	Uncertainty and data	Formulate
Stop The Car	PM571Q01	44.03	47.68	53.06	56.11	57.65	58.10	63.80	Change and relationships	Interpret
Number Check	PM603Q01T	39.30	45.08	47.51	57.76	44.68	47.06	59.42	Quantity	Employ
Computer Game	PM800Q01	79.49	88.41	87.28	84.64	88.33	96.45	96.88	Quantity	Employ
Labels	PM803Q01T	25.78	29.19	36.01	36.06	44.29	43.23	45.89	Uncertainty and data	Formulate
Carbon Dioxide	PM828Q01	29.87	28.48	38.32	33.09	18.20	49.06	62.09	Change and relationships	Employ
Carbon Dioxide	PM828Q02	58.68	55.97	62.50	59.80	69.44	66.28	76.04	Uncertainty and data	Employ
Carbon Dioxide	PM828Q03	21.10	28.03	31.55	36.65	37.28	14.19	52.42	Quantity	Employ
Drip rate	PM903Q01	19.24	22.24	24.69	27.71	24.09	43.32	70.67	Change and relationships	Employ
Drip rate	PM903Q03	29.92	25.72	37.09	24.79	30.62	46.27	70.77	Change and relationships	Employ
Tennis balls	PM905Q01T	78.25	77.71	82.53	79.57	82.84	86.18	86.18	Quantity	Interpret
Tennis balls	PM905Q02	38.70	50.07	57.38	56.35	61.69	63.12	80.62	Quantity	Employ
Crazy ants	PM906Q01	50.70	60.67	62.11	67.66	75.23	61.07	79.18	Quantity	Employ
Crazy ants	PM906Q02	42.93	42.15	56.73	49.03	50.76	59.18	84.05	Quantity	Employ
Speeding fines	PM909Q01	92.91	89.35	94.40	92.44	95.72	93.51	94.43	Quantity	Interpret
Speeding fines	PM909Q02	56.32	63.14	68.04	72.49	77.16	71.25	74.25	Quantity	Employ
Speeding fines	PM909Q03	28.21	35.71	40.01	44.93	47.43	55.00	62.10	Change and relationships	Interpret
Carbon tax	PM915Q01	43.56	40.20	40.72	31.91	49.44	66.66	72.63	Uncertainty and data	Employ
Carbon tax	PM915Q02	52.94	68.26	76.10	80.46	75.40	74.02	90.10	Change and relationships	Employ
Charts	PM918Q01	91.93	87.27	86.89	87.95	90.43	90.76	92.55	Uncertainty and data	Interpret
Charts	PM918Q02	76.97	79.56	82.55	85.19	83.47	86.88	89.70	Uncertainty and data	Interpret
Charts	PM918Q05	76.66	76.68	83.58	77.00	81.88	85.05	89.04	Uncertainty and data	Employ
Z's fan merchandise	PM919Q01	78.76	84.53	85.81	88.68	86.30	89.06	94.13	Quantity	Employ
Z's fan merchandise	PM919Q02	39.28	44.73	49.38	46.86	49.51	52.37	56.01	Quantity	Formulate

Table A1.1 [Part 2/2]

Items by solution rates for United States and comparator countries

		Solution rates (percentage of students providing the correct answer)							Framework categories	
Name	Item Code	United States	OECD average	Canada	Germany	Netherlands	Korea	Shanghai-China	Content	Process
Sailing ships	PM923Q01	50.09	59.50	58.03	64.10	78.08	69.10	73.91	Quantity	Employ
Sailing ships	PM923Q03	43.69	49.81	57.49	53.59	61.97	56.27	81.84	Space and shape	Employ
Sailing ships	PM923Q04	11.48	15.28	20.72	19.61	24.83	21.16	47.04	Change and relationships	Formulate
Sauce	PM924Q02	51.32	63.48	60.64	66.49	81.32	73.22	85.13	Quantity	Formulate
Arches	PM943Q01	51.17	50.04	53.80	52.01	45.06	62.81	78.57	Change and relationships	Formulate
Arches	PM943Q02	2.48	5.29	4.14	5.82	3.25	19.51	50.93	Space and shape	Formulate
Roof truss design	PM949Q01T	57.68	67.54	75.97	73.06	77.77	85.38	89.75	Space and shape	Formulate
Roof truss design	PM949Q02T	27.81	31.75	38.49	26.59	21.14	73.62	86.04	Space and shape	Employ
Roof truss design	PM949Q03	31.45	32.57	35.93	32.00	39.54	68.39	83.20	Space and shape	Formulate
Flu test	PM953Q02	48.70	49.78	61.13	51.83	47.50	63.03	74.27	Uncertainty and data	Interpret
Flu test	PM953Q03	44.29	51.82	56.27	56.60	63.86	67.13	81.34	Uncertainty and data	Formulate
Flu test	PM953Q04D	16.52	18.22	24.74	20.22	23.67	35.38	53.08	Uncertainty and data	Formulate
Medicine doses	PM954Q01	70.33	65.42	71.93	68.42	58.88	81.18	92.45	Change and relationships	Employ
Medicine doses	PM954Q02	31.73	33.57	36.89	44.77	35.36	40.04	56.23	Change and relationships	Employ
Medicine doses	PM954Q04	23.54	26.36	33.00	36.01	32.63	38.76	54.49	Change and relationships	Employ
Migration	PM955Q01	73.54	72.13	79.06	79.03	70.39	75.15	84.35	Uncertainty and data	Interpret
Migration	PM955Q02	19.61	34.23	34.95	36.35	48.47	56.27	57.59	Uncertainty and data	Interpret
Migration	PM955Q03	5.97	11.99	14.97	14.35	22.29	23.96	42.90	Uncertainty and data	Employ
Employment data	PM982Q01	87.23	87.30	87.76	89.08	88.66	85.34	81.65	Uncertainty and data	Employ
Employment data	PM982Q02	33.19	30.74	35.81	28.72	30.06	44.53	37.59	Uncertainty and data	Employ
Employment data	PM982Q03T	58.54	64.96	64.97	69.84	70.77	73.80	73.29	Uncertainty and data	Interpret
Employment data	PM982Q04	43.32	51.46	51.98	60.01	66.01	58.89	68.10	Uncertainty and data	Formulate
Spacers	PM992Q01	75.45	77.61	84.69	83.29	79.63	86.78	90.98	Space and shape	Formulate
Spacers	PM992Q02	14.20	18.26	23.55	16.53	15.63	40.50	45.54	Space and shape	Formulate
Spacers	PM992Q03	5.43	8.11	8.72	9.67	6.87	27.34	38.07	Change and relationships	Formulate
Revolving door	PM995Q01	46.96	57.71	57.44	63.99	59.09	79.08	89.70	Space and shape	Employ
Revolving door	PM995Q02	2.26	3.47	4.26	3.32	4.09	6.35	13.62	Space and shape	Formulate
Revolving door	PM995Q03	44.62	46.44	54.21	49.38	56.58	52.65	65.20	Quantity	Formulate
Bike rental	PM998Q02	80.71	71.59	84.93	76.06	89.07	80.06	87.63	Change and relationships	Interpret
Bike rental	PM998Q04T	32.26	40.45	36.68	45.42	43.40	49.90	57.15	Change and relationships	Employ

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

The OECD is a unique forum where governments work together to address the economic, social and environmental challenges of globalisation. The OECD is also at the forefront of efforts to understand and to help governments respond to new developments and concerns, such as corporate governance, the information economy and the challenges of an ageing population. The Organisation provides a setting where governments can compare policy experiences, seek answers to common problems, identify good practice and work to co-ordinate domestic and international policies.

The OECD member countries are: Australia, Austria, Belgium, Canada, Chile, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, the Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The European Union takes part in the work of the OECD.

OECD Publishing disseminates widely the results of the Organisation's statistics gathering and research on economic, social and environmental issues, as well as the conventions, guidelines and standards agreed by its members.

OECD PUBLISHING, 2, rue André-Pascal, 75775 PARIS CEDEX 16
(98 2013 03 1P1) ISBN 978-92-64-20757-8 – No. 61103 2013

Strong Performers and Successful Reformers in Education
Lessons from PISA 2012 for the United States

In 2010, U.S. President Barack Obama launched one of the world's most ambitious education reform agendas. Entitled "Race to the Top", this initiative encourages U.S. states to promote innovation, reform and excellence in public schools. Three years on from the launch of Race to the Top, where does the United States stand when compared internationally? Will effective implementation of the Common Core State Standards improve the performance of the United States internationally? The OECD Programme for International Student Assessment (PISA) provides the world's most extensive and rigorous set of international surveys of the knowledge and skills of secondary school students.

This report situates the PISA performance of 15-year-olds in the United States against global patterns and trends. In addition, it aims to go beyond aggregate-level analysis to examine student performance on individual mathematics test items and reveal students' strengths and weaknesses.

Contents

Further reading

PISA 2012 Results - Volumes I to IV (2013)

Strong Performers and Successful Reformers in Education: Lessons from PISA for the United States (2011)

Consult this publication on line at: ***http://dx.doi.org/10.1787/9789264207585-en***

This work is published on the *OECD iLibrary,* which gathers all OECD books, periodicals and statistical databases. Visit ***www.oecd-ilibrary.org*** and do not hesitate to contact us for more information.

2013

OECD*publishing*
www.oecd.org/publishing

ISBN 978-92-64-20757-8
98 2013 03 1P1

9 789264 207578